Replace page 4 of Solutions03.pdf with the following correction:

[boundaries]

for $q \geq 2$ and $B_{q+1}(K) \cong B_{q+1}(K_1) \oplus B_{q+1}(K_2)$ for $q \geq 1$. It remains to check that $Z_1(K) = Z_1(K_1) \oplus Z_1(K_2) \subseteq C_1(K_1) \oplus C_1(K_2) \cong C_1(K)$.

Suppose $(a_1, a_2) \in C_1(K_1) \oplus C_1(K_2)$ is a cycle, so $d_1 a_1 + d_1 a_2 = 0$, or $da_1 = -da_2$. Since $C_0(K_1) \cap C_0(K_2)$ is the subgroup generated by the vertex $z \in P_1 \cap P_2$ and $da_i \in C_0(K_i)$, we must have $d_1 a_1 = pz = -d_1 a_2$ for some $p \in \mathbb{Z}$. Suppose now that we apply the augmentation $\varepsilon : C_0(K) \to \mathbb{Z}$. Then $\varepsilon d_1 = 0 \Rightarrow 0 = \varepsilon d_1 a_1 = \varepsilon(pz) = p$, which means that $da_1 = 0 = da_2$. Thus $Z_1(K) = Z_1(K_1) \oplus Z_1(K_2)$, which is what we wanted. ∎

Comment: One can derive this result far more easily using the Mayer-Vietoris sequence for $(K; K_1, K_2)$.