## Star shaped simplicial complexes

A simplicial complex ( $\boldsymbol{P}, \mathbf{K}$ ) is star shaped with respect to a vertex $\mathbf{v}$ if $\boldsymbol{P}$ is the union of all simplices in $\mathbf{K}$ which have $\mathbf{v}$ as one of their vertices. An equivalent formulation of this concept is that every simplex in $\mathbf{K}$ is a face of another simplex which has $\mathbf{v}$ as one of its vertices.

It follows that a simplex is star shaped with respect to each of its vertices; conversely, if a simplicial complex is star shaped with respect to each vertex, then it is a simplex. (Sketch of proof: This is obvious if the complex has two vertices; proceed by induction on the number of vertices). Here are two examples of convex polygonal regions with simplicial decompositions that are star shaped with respect to one of the vertices.


And here is an example of a polygonal region which is star shaped with respect to the inner vertex but is not convex.


