Mathematics 205B, Winter 2018, Assignment 1

This will be due on **Monday, January 29, 2018, at 11:10 A.M.** at the beginning class or by prior arrangement in my mailbox or at the front desk of Surge 202 at the same time. If you wish to use some version of TEX in writing up your answers, please feel free to do so.

You must show the work behind or reasons for your answers.

1. Let X be a space which is Hausdorff, connected and locally arcwise connected, and let $f : (X, x) \to (S^1, 1)$ be a continuous mapping. Prove that f has a square root (a continuous mapping $gX \to S^1$ such that $g(x)^2 = f(x)$, where the squaring operation comes from complex multiplication) if and only if the image of $f_* : \pi_1(X, x) \to \pi_1(S^1, 1) \cong \mathbb{Z}$ is contained in $2 \cdot \mathbb{Z}$. [*Hint:* Recall that the the squaring map q from S^1 to itself is a basepoint preserving covering space projection.]

2. In the lectures there is a classification of arcwise connected Hausdorff covering spaces over a given space. The point of this exercise is to prove a generalization for Hausdorff covering spaces which are not necessarily arcwise connected.

(a) Let B be a space which is Hausdorff, locally arcwise connected and connected, and let $b_0 \in B$. Suppose further that $p : (E, e_0) \to (B, b_0)$ is a covering space projection such that E is Hausdorff (note that it is automatically locally arcwise connected). If E_0 is the arc component of e_0, prove that $P|E_0$ is also a covering space projection.

(b) Let $p : (E, e_0) \to (B, b_0)$ be as in (a). Prove that E is a union of pairwise disjoint open closed subspaces E_α such that each E_α is arcwise connected and each restriction $p|E_\alpha$ is a covering space projection.

(c) Let $p : (E, e_0) \to (B, b_0)$ be as in (a) once more, and now assume that B is simply connected. Prove that E is homeomorphic to $B \times F$, where F is the fiber $p^{-1}[\{b_0\}]$. [*Hint:* Why is each restriction $p|E_\alpha$ a homeomorphism?]