Triangulations of polygonal regions in the plane

The first diagram shows a simple polygon, albeit a rather convoluted one.



http://www.ams.org/samplings/feature-column/fcarc-diagonals4

This polygon can be triangulated in a variety of ways, one of which is shown below. Observe that the vertices of the triangulation are just the vertices of the broken line curve which bounds the region; in other words, it is a triangulation with no extra vertices.



Different subsets of the plane may have isomorphic triangulations. In the drawing below, the isomorphism is uniquely determined by its effect on vertices: The vertices **a**, **b**, **c**, **d**, and **e** correspond to **c'**, **b'**, **a'**, **e'**, and **d'** respectively.



Here is a more complicated example of a triangulation.



http://people.sc.fsu.edu/~jburkardt/cpp_src/triangulation/triangulation_order3_plot.png

And here are still others.



http://en.wikipedia.org/wiki/File:Triangulation 3-coloring.svg



http://mathworld.wolfram.com/Triangulation.html

Triangulating 3 – dimensional polyhedra

We need some way of triangulating 3 – dimensional polyhedra, and of course this must work for the following standard examples:



http://hotmath.com/hotmath_help/topics/surface-area-of-a-prism.html



Egyptian pyramid (square base)

http://www.sacred-geometry.com/chi-ometry/Giza_Pyramid.htm

It turns out that the right sorts of subsets for 3 – dimensional triangulations are pyramids with triangular bases. For example, if we decompose the base of a square pyramid into four triangular regions as in the following drawing



then we obtain an associated decomposition of the Egyptian pyramid into four pyramids with triangular bases:



In other files we shall indicate how one can decompose the closed region bounded a triangular prism into three closed regions bounded by triangular pyramids. If we combine this with the decompositions of polygonal regions in the plane into unions of triangular regions in the plane, we obtain decompositions of closed regions bounded by arbitrary prisms into regions bounded by triangular pyramids. We shall also give some references for similar decompositions of more general objects in the notes.