## UPDATED GENERAL INFORMATION — MARCH 7, 2014

*Review for the second examination*

The examination will cover Units IV–VI and Sections VII.1–VII.4 in `algtop-notes.pdf` (along with the corresponding sections in Munkres and Hatcher listed at the beginning of these section), with the following exceptions: Sections V.$\infty$ and VI.$\infty$ are not included in the coverage, and the coverage of Section VII.4 will only go through the end of the discussion about the utilities network on page 98.

**Files of exercises.** The files starting with `exercises03-2012.pdf` correspond to the material on the examination. Here are some recommendations for exercises that should have high priorit, exercises that should have low priority, and exercises whose solutions should be read through in order to get a passive understanding of the solutions.

(**03**) *High priority:* 1, 2($b$), 5, 8, 9, 11, 13, 14. *Low priority:* 4, 12. *Read:* 6, 7, 10.

(**04**) *High priority:* 0($c$), 4, 5, 6, 7, 11. *Low priority:* 10. *Read:* 9.

(**05**) *High priority:* 4, 5, 6, 7, 11. *Low priority:* 7, 8, 9, 10, 12. *Read:* 1, 2, 3, 12.

**Additional exercises.** The following also deserve high priority:

**1.** Let $\beta_1, \cdots, \beta_n$ denote a sequence of nonnegative integers. Find an $n$-dimensional simplicial complex $\mathbf{K}$ such that $H_q(\mathbf{K}\ etc.)$ is free abelian of rank $\beta_q$ for $1 \leq q \leq n$. Also, what are $H_0$ and $H_{n+1}$?

**2.** Let $f : D^n \to \mathbb{R}^n$ be continuous and 1–1. Prove that $f$ maps the interior of $D^n$ onto the interior of $h[D^n]$.

**3.** Suppose we have a sequence of abelian groups $A \to B \to C$ such that $A \to B$ and $B \to C$ are isomorphisms. Prove that $A = B = C = 0$.

**Old examinations on file.** As before, examinations from the Winter 2012 version of this course are in the course directory, and the folder `old-exams` in this directory also contains earlier examinations (in class and take home) which may be useful for review.

**Priorities for knowing and understanding.** In Unit IV, the main concepts are the notion of a simplex and its faces, a simplicial complex, a chain complex, the homology of a chain complex, and an exact sequence. The basic definitions should be known as well as the proofs of some consequences (typical level: If a map of chain complexes is 1–1 and onto, then the inverse homomorphisms of chain groups determine a map of chain complexes). In Unit V, the main concept is the notion of homology for a simplicial complex with a linear ordering of the vertices. Also important are the relative groups for pairs of complexes, the long exact homology sequence for a pair (the derivation should be understood passively), the homology groups of a point and more generally a star-shaped complex, the excision result for relative chains and homology, and Mayer-Vietoris sequence associated to a decomposition of a complex as a union of two subcomplexes. Some familiarity with uses of these techniques for computational pruposes is also important. In Unit VI, the main things are to understand the axioms for an abstract singular homology theory, which center around functoriality properties, the long exact sequence of a pair, homotopy invariance, compact support, normalization, the excision result for spaces which are the union of two open subsets, and the Mayer-Vietoris sequence for such decomposed spaces. Once again, some computational familiarity is desirable. Finally, for the applications in Unit VII the main definition is that of local homology at a point, accompanied by the use of excision to show that local homology only depends upon the topological structure in a small neighborhood of a point. The main theorems to know include invariance of dimension, the application of local homology to show that certain graphs are not homeomorphic, how to show that a subspace is not a retract of a larger space containing it, the statement of the Brouwer Fixed Point Theorem, and the statements of the separation theorems for complements of disks, spheres and graphs. More precise information will be supplied early next week after the exam has been written.