NAME: ______________________________

Mathematics 205B, Winter 2008, Final Examination

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers.

#	SCORE
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TOTAL	

1. [*20 points*] Let $K \subset \mathbf{R}^n$ be a nonempty convex set, and let $k_0 \in K$. Prove that $\{k_0\}$ is a strong deformation retract of K.

2. [*20 points*] Prove that S^1 is not a retract of S^n if $n > 1$.

3. [*25 points*] Let $p : E \to B$ be a covering space projection such that E is compact Hausdorff. Prove that p has finitely many sheets.

4. [*25 points*] Let $\Gamma \subset S^2$ be a simple closed curve, and let $x,\ y,\ z$ be three distinct points of S^2 which do not lie on Γ. Prove that one can find a pair of points $u, v \in \{x, y, z\}$ that can be joined by a continuous curve which does not meet Γ. [*Hint:* If three points lie in $A \cup B$ and $A \cap B = \emptyset$, why do at least two lie in one of A or B?]

5. [*25 points*] Let G be a finite group of order n. Prove that G is isomorphic to a homomorphic image of the free group F_{n-1} on $n-1$ generators. [*Hint:* The nontrival elements of the group can be listed as $g_1, \cdots, g_{n-1}$.]

Optional extra credit: [*15 points*] If G has odd order $2k+1$, show that G is a homomorphic image of a free group on k generators.

6. [*30 points*] Let X be a Hausdorff, connected, locally simply connected space which is a union of two connected open subsets U and V such that $U \cap V$ is also connected. Suppose further that the standard maps from $\pi_1(U \cap V)$ to both $\pi_1(U)$ and $\pi_1(V)$ are surjective. Prove that the standard map from $\pi_1(U \cap V)$ to $\pi_1(X)$ is also surjective.

7. [*25 points*] Let $f : (X, x_0) \to (Y, y_0)$ be a base point preserving continuous map of connected, Hausdorff, locally simply connected spaces, and let $p : (\widetilde{X}, x_1) \to (X, x_0)$ and $q : (\widetilde{Y}, y_1) \to (Y, y_0)$ be universal (simply connected) covering space projections. Prove that there is a unique base point preserving continuous mapping $g : (\widetilde{X}, x_1) \to (\widetilde{Y}, y_1)$ such that $p \circ g = q \circ f$.

8. [*30 points*] Let X be the pentagram graph as depicted on the accompanying sheet, and take the decomposition into 15 edges as suggested by the figure.

(i) If T is a maximal tree in X, how many edges are in T?

(ii) Describe the fundamental group of X in abstract group-theoretic terms, and give reasons for your answer.