NAME: ______________________________

Mathematics 205B, Winter 2021, Examination 1

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Please submit a copy of your exam by electronic mail to the following address by 11:59 P.M. on Friday, February 19, 2020:

`rschultz@ucr.edu`

Each problem should be started on a separate sheet of paper. Make sure your submission is readable; some smartphone photos might not have enough contrast. You may look at outside references such as course directory documents before starting this examination, and you may consult with other students, the teaching assistant or me about material related to this examination, but this assignment is **NOT** collaborative. The answers you submit must be your own work and nobody else's.

The top score for setting the curve will be 100 points.

#	**SCORE**
1	
2	
3	
4	
TOTAL	

1. [*25 points*] Suppose that X is the union of the arcwise connected open sets $U \cup V$ such that $U \cap V$ is also arcwise connected. Assume further that the homomorphismss $\pi_1(U \cap V) \to$ (*both*) $\pi_1(U),\ \pi_1(V)$ induced by inclusion are isomorphisms. Prove that the maps from both $\pi_1(U)$ and $\pi_1(V)$ to $\pi_1(X)$ induced by inclusion are also isomorphisms.

2. [*25 points*] (a) If the following statement is true, give a proof; if it is false, give a counterexample: *Suppose that the graph T is a tree and F is an edge in T. Then the subgraph formed by all edges except F is a tree.*

(b) Give an example of a connected graph with exactly one maximal tree, and give an example of a connected graph with at least two maximal trees.

3. [*25 points*] Let W be a simply connected space, let X be a connected graph, and let $Y \to X$ be a finite covering. Prove that every continuous map $W \to X$ lifts to a continuous mapping from W to Y. You may assume all spaces are also Hausdorff and locally arcwise connected.

4. [*25 points*] Suppose that we are given a solid regular n-gon X in the plane, where $n \geq 4$, and let $v_1, \cdots, v_n$ are its vertices so that the boundary is the closed path $x_1x_2 + x_2x_3 + ... + x_nx_1$. Assume the vertices are ordered as indicated; denote this ordering by ω. Let $\mathbf{K}$ be the triangulation of X by the 2-simplices $x_ix_{i+1}x_n$ where $i = 1, ..., n-2$. A drawing is given on the next page. Find a simplicial 2-chain in $C_2(X, \mathbf{K}, \omega)$ whose boundary is a sum of terms

$$\varepsilon_1 x_1 x_2 \ + \ \varepsilon_2 x_2 x_3 \ + \ \cdots \ \varepsilon_{n-1} x_{n-1} x_n \ + \ \varepsilon_n x_1 x_n$$

where $\varepsilon_i = \pm 1$. Prove that the boundary of your 2-chain has the required property.

Drawing for Problem 4