NAME: ______________________________

Mathematics 205B, Winter 2021, Examination 2

INSTRUCTIONS: Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets. Please submit a copy of your exam by electronic mail to the following address by **11:59 P.M.** on **Wednesday, March 17, 2021:**

`rschultz@ucr.edu`

Each problem should be started on a separate sheet of paper. Make sure your submission is readable; some smartphone photos might not have enough contrast. You may look at outside references such as course directory documents before starting this examination, and you may consult with other students or me about material related to this examination, but this assignment is **NOT** collaborative. The answers you submit must be your own work and nobody else's; this includes use of mechanical or electronic computational devices.

The top score for setting the curve will be 125 points.

Please make sure that an electronic copy of your completed exam is also sent to your email account in case there are unexpected transmission problems.

1. [*25 points*] Let $\Delta_2 \times I$ denote the standard solid 3-dimensional triangular prism, with ordered vertices $\mathbf{x}_0$, $\mathbf{x}_1$, $\mathbf{x}_2$ (bottom) and $\mathbf{y}_0$, $\mathbf{y}_1$, $\mathbf{y}_2$ (top); the boundary $\partial(\Delta_2 \times I) = (\Delta_2 \times \{0,1\}) \cup (\partial\Delta_2 \times I)$ is then a subcomplex of the given simplicial decomposition. Find a chain $A \in C_2((\Delta_2 \times I), \text{decomp}, \omega)$ such that A is a linear combination of every 2-simplex in ∂P, the coefficient of each free generator is $\pm\, 1$, and $d_2(A) = 0$. **(*P* = the prism)**

2. [*25 points*] (a) Show that $(\mathbb{R}^2 \times \{0\}) \cup (\{0,0\} \times \mathbb{R}) \subset \mathbb{R}^3$ is not homeomorphic to $\mathbb{R}^2$. [*Hint:* Look at local propeties at the origin. Any valid method is acceptable.]

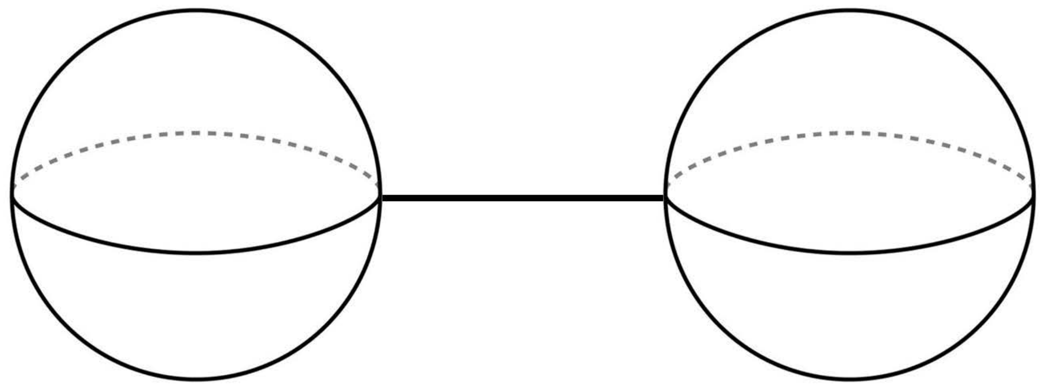
(b) Show that the standard inclusion $S^1 \times S^1 \subset S^3$ is not a retract.

3. [*25 points*] Suppose that A is a nonempty subspace of the topological space X, and let $i : A \to X$ denote the inclusion. Prove that all the maps in homology $i_* : H_q(A) \to H_q(X)$ are isomorphisms if and only if all of the relative homology groups $H_q(X, A)$ are trivial. [*Hint:* What does it mean to have zero mappings in an exact sequence?]

4. [*25 points*] (*a*) Suppose we are given a subset $A \subset S^3$ which is a union of three compact subsets $B_1 \cup C \cup B_2$ where B_1 and B_2 are disjoint subsets which are homeomorphic to S^2 and C is homeomorhic to a closed interval such that each intersection $C \cap B_i$ is an endpoint of C (see the next page for a drawing). Prove that the complement $S^3 - A$ has three connected components. [*Hint:* What is the reduced homology of $S^2 - (B_i \cup C)$ for $i = 1, 2$?] **(Replace S^2 with S^3 in the hint.)**

EXTRA CREDIT. [*10 points*] For each component Ω as above, state a conjecture about which points of A should be limit points of Ω.

Drawing for Problem 4



5. [*25 points*] Let U and V be (arcwise) connected open subsets of $\mathbb{R}^n$ such that $U \cap V$ is a nonempty convex set and $\pi_1(U \cap V)$ is finite. Prove that at least one of U and V must be simply connected. [*Hint:* What is the contrapositive?]

Correction: **Change π_1(intersection) to π_1(union).**