NAME: ______________________________

Mathematics 205B, Winter 2019, Examination 1

Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets.

Assume all spaces are Hausdorff and locally arcwise connected.

#	SCORE
1	
2	
3	
4	
TOTAL	

1. [*25 points*] Let X be an arcwise connected space such that $X = U_1 \cup U_2$, where each U_i is open and arcwise connected and $U_1 \cap U_2$ is also arcwise connected. Denote the inclusions of the intersection into the U_i by q_i. Assume further that $\pi_1(U_1 \cap U_2) \cong \mathbb{Z} \times \mathbb{Z}$, $\pi_1(U_i) \cong \mathbb{Z}$ (both i), and the induced homomorphisms $q_{i*} : \pi_1(U_1 \cap U_2) \to \pi_1(U_i)$ correspond to the coordinate projections from $\mathbb{Z} \times \mathbb{Z}$ to $\mathbb{Z}$. Prove that $\pi_1(X)$ is trivial.

2. [*25 points*] One of the following statements is true and the other one is false. State which one is true and give reasons for your answer. There is no need to give a counterexample to the false statement.

(1) If $U \subset \mathbb{R}^n$ is a nonempty open and arcwise connected subset, then U has a simply connected covering space.

(2) If $F \subset \mathbb{R}^n$ is a nonempty closed and arcwise connected subset, then F has a simply connected covering space.

3. [*25 points*] (*a*) Let $(X, \mathcal{E})$ be a connected graph, and let $Y \subset X$ be a connected nonempty subgraph. Prove that if X is a tree, then Y is also a tree.

(*b*) Let $(X, \mathcal{E})$ be a connected graph with V vertices. Prove that the number E of edges is less than or equal to $f(V)$ for some quadratic polynomial f, and give a specific example of such a polynomial. Recall that an edge is determined by its endpoints.

4. [*25 points*] Let $X \subset \mathbb{R}^2$ be the union of the two closed curves defined by the equations $|x| + |y| = 3$ and $\max\{|x|, |y|\} = 2$. One can then define a connected graph complex structure on X which is symmetric with respect to both coordinate axes. Find all the vertices of one such structure which lie in the first quadrant ($x, y \geq 0$), and find the positive integer k such that $\pi_1(X)$ is a free group on k generators.

Extra page for use if needed