**NAME:** ______________________________

# Mathematics 205B, Winter 2018, Examination 1

*Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets.*

| # | SCORE |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| TOTAL | |

1. [35 *points*] These questions involve the pentagram (5–pointed star) graph on the next page.

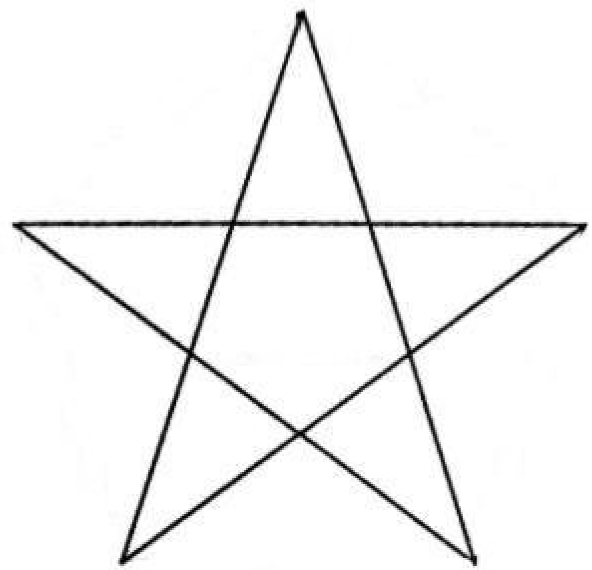
($a$) The fundamental group of this graph is a free group on finitely many generators. Determine the number of such generators.

($b$) Sketch a maximal tree in this graph.

($c$) Describe an Euler path for this graph.

There is space for drawings on this page and the next one.

# Drawing for Problem 1



**Source: http://images.huffingtonpost.com/2014-11-20-pentagram.jpg**

2. [25 *points*] Let $p : (E, e) \to (S^1, 1)$ be a connected covering space and assume $E$ is compact Hausdorff. Prove that $E$ is homeomorphic to $S^1$. [*Hint:* For each $n$ find an $n$-sheeted covering space projection onto $S^1$.]

3. [*20 points*] Suppose that $X$ and $X'$ are subspaces in $\mathbb{R}^n$ such that there are graph structures $\mathcal{E}$ and $\mathcal{E}'$ on $X$ and $X'$ respectively such that $X \cap X'$ is a single point which is a vertex of both. Explain why $X \cup X'$ has a graph complex structure.

4. [20 *points*] Suppose that $X$ is the union of the open arcwise connected subspaces $U$ and $V$ such that $U \cap V \neq \emptyset$ is arcwise connected and both $U$ and $X$ are simply connected; take a base point for all of these (sub)spaces which lies in $U \cap V$. Prove that if $H$ is the normal subgroup $\pi_1(V)$ which is (normally) generated by the image $\pi_1(U \cap V)$, then $H = \pi_1(V)$.

Extra page for use if needed