Winter 2012

Review Suggestions for the Math 205B Final Examination

50% of the exam will cover material in Units I – III, and the earlier suggestions in review1.pdf still apply (⟹ will not be repeated)

1. Understand the definitions and basic properties of simplicial complexes (including background, like notion of a k-simplex). In particular, understand criteria for recognizing connected complexes in terms of the simplices. Know the basic method for finding a simplicial decomp. of $P \times [0, 1]$ if (P, K) is a simplicial complex.

2. Know the basic definitions and algebraic properties of abstract chain complexes and their homology groups, including maps of chain complexes, associated homology groups, long exact homology sequences associated to short exact sequences of chain complexes.

3. Know the definition of the simplicial chain complex associated to a simplicial complex and the basic results about simplicial homology: The groups are finitely generated abelian groups, they vanish above dim K and in negative dims, $H_0 \cong$ free abelian on components, star-shaped $\Rightarrow$ same homology as Δ_0, long exact sequences of pairs and Mayer-Vietoris sequences, the excision property and its proof (same for M-V), proof that $d \circ d = 0$ in the definition of the simplicial chain complex, computation of $H_*(\partial \Delta_{n+1})$.

4. Know the basic axioms for singular homology; at least qualitatively: (1) It expands upon simplicial homology. (2) Functorial for arbitrary cont. maps. (3) Homotopy invariance (4) Compact supports, $H_*(X) \cong \bigoplus H_*(X_\alpha)$ where X_α runs through the arc components, $H_0(\text{arc. conn.}) \cong \mathbb{Z}$ (5) Excision and M-V sequences. Know how to work from axioms, or something which is given, to prove spaces are not homeomorphic, nonretraction results (e.g., S^2 is not a retract of S^3).

5. Know how to work with applications. For example, use known results to prove other things like: "If $f: S^n \to S^n$ s.t. $f_*: H_n(S^n) \to H_*(S^n)$ is nonzero, then f is onto." Also know the def and

basic properties of local homology groups at a point.

Rejected problem: Let $\mathbb{R}^n_+$ = all pts in $\mathbb{R}^n$ whose last coord is ≥ 0. Find $H_*(\mathbb{R}^n_+, \mathbb{R}^n_+ - \{0\})$.

HINT Let D^{n-1}_+ = all pts in $\mathbb{R}^n_+$ s.t. $|x| = 1$. Why is $\mathbb{R}^n_+ - \{0\} \cong (0,\infty) \times D^n_+$?

Of course one should also know what $H_*(S^n)$ is and how this relates to simplicial homology ($D^n \cong [0,1]^n \cong$ simplex, etc.).

> There will be no specific questions on stating definitions, theorems, etc, but parts of several problems will require knowledge of such items.

5.' Know statements of basic applications like the Jordan-Brouwer Separation Thm., the Brouwer Fixed Pt. Thm., Invariance of Dimension, Invariance of Domain.