Math 205B
Winter 2018

UPDATED GENERAL INFORMATION — FEBRUARY 9, 2018

Review for midterm examination

The examination will cover material through Section IV.1 in the course notes. Exercises and reading assignments for Units I–III have already been posted. Here are additional items for Section IV.1.

First of all, the coverage will include material in Sections V.2–V.4 in `math145Bnotes5.pdf` and the related exercise files.

Working the exercises listed below is **strongly recommended.**

The following exercise is taken from `fundgp-exercises.pdf`:

IV.3.4

The following exercises are taken from `math145Bexercises5s15.pdf`:

V.2.1, 2, 4, 5

V.4.1–3

Readings for Section IV.1

In addition to the usual files and the one named above, here are further recommendations:

`koenigsberg.pdf`
`koenigsberg-sequel.pdf`

Additional information and illustrations for the Königsberg Bridge Problem.

`math145Bsolutions5s15.pdf`

Predictably, solutions to the 145B exercises.

One particularly important point is to know the definitions thoroughly and to understand how they can be applied in arguments. In many cases, knowing the definitions will get you well into the solutions.

Practice problems

1. Let X be a simply connected Hausdorff space which is also locally arcwise connected. Prove that every simply connected covering space of $X \times S^1$ is homeomorphic to $X \times \mathbb{R}$.

2. Let X be a connected compact Hausdorff space which is also locally arcwise connected, and assume that X has a compact simply connected covering space. Prove that every continuous mapping $X \to S^1$ is homotopic to a constant map. [*Hint:* What can we say about $\pi_1(X)$?]

3. Let $X = U \cup V$ where U, V and $U \cap V$ are all nonempty and arcwise connected. Pick a base point in $U \cap V$, and assume that $\pi_1(U \cap V) \cong \mathbb{Z} \times \mathbb{Z}$ and $\pi_1(U) \cong \mathbb{Z}$ such that the map $\pi_1(U \cap V) \to \pi_1(U)$ corresponds to the homomorphism $\mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by projection onto the first factor. Prove that the image of $(0,1) \in \mathbb{Z} \times \mathbb{Z} \cong \pi_1(U \cap V)$ in $\pi_1(V)$ normally generates the latter.

4. Suppose that the topological space X is a union of the arcwise connected open subspaces U and V where $U \cap V$ is also arcwise connected, and let $p \in U \cap V$. Suppose further that the map of fundamental groups $\pi_1(U \cap V, p) \to \pi_1(V, p)$ corresponds to an inclusion mapping $H \to G$ where H is a normal subgroup of G, and also that V is simply connected. Prove that $\pi_1(X, p)$ is isomorphic to G/H.

5. Which of these is not homeomorphic to a 2-sheeted covering space of $S^1 \times \mathbb{RP}^2$?

$S^1 \times \mathbb{RP}^2$
$S^1 \times S^2$
$S^1 \times T^2$ (where $T^2 = S^1 \times S^1$)

6. Suppose that X is the underlying space of a connected graph whose fundamental group is free on 9 generators, and let $W \to X$ be a connected 2-sheeted covering. Then the fundamental group of W is free on N generators for some nonnegative integer N. Find N, giving reasons for your answer.

7. Let X be a connected graph which has Euler paths, and suppose that P_1 and P_2 are reduced Euler paths in X. Why do these paths have the same numbers of edges? Give an example in which there are at least two Euler paths.

8. Suppose we have two finite graph complexes X_1 and X_2 which lie in some larger space Y such that $X_1 \cap X_2$ is the set of vertices for each of the graphs. Show that $X_1 \cup X_2$ has a graph complex structure. [*Hint:* It might be necessary to subdivide edges. Think of the case where X_1 and X_2 consist of a single edge.]

9. Prove a similar result if X_1 and X_2 only meet in a single vertex of both. [*Hint:* Is it necessary to subdivide anything here?]