Math 205B
Winter 2018

UPDATED GENERAL INFORMATION — MARCH 1, 2018

Readings for Sections IV.2 – IV.4

In addition to `algtop-notes.pdf` and the corresponding exercise and solutions, here are some recommendations. An asterisk "$*$" in a file name denotes a wild card; for example, `part*.pdf` might denote files `part1.pdf` and `part2.pdf`, and similarly `filename.*` may denote different types of files with the same basic name.

`three-simplex.pdf`

Drawings of 3-dimensional simplices (= solid tetrahedra).

`triangulation-pictures.pdf`
`triangulations*.pdf`

Examples of spaces which are homeomorphic to other spaces that admit simplicial decompositions, together with drawings and written remarks.

`centroids.pdf`

A fairly elementary derivation of the classical formula for the center of mass of an object.

`barycentric.pdf`

A drawing for a (barycentric) decomposition of a 2-simplex.

`convex*.pdf`

Drawings of some 3-dimensional polyhedra, all of which have simplicial decompositions.

`prism-dissection.pdf`

Drawings of a simplicial decomposition for a 3-dimensional triangular prism.

`starshaped.pdf`

Drawings of 2-dimensional starshaped simplicial complexes.

`moebius*.*`

Files with drawings of the Möbius strip, including triangulations of the latter and animated illustrations of its one-sidedness.

`rp2triangulation.pdf`

Drawing of a triangulation for the real projective plane.

`prismatoids.pdf`

An application of simplicial decompositions to derive a classical volume formula in solid geometry.

`massey-chapter6.pdf`

An excellent discussion of ideas underlying the fundamental constructions in algebraic topology.

`wsw*.pdf`

A very challenging set of problems designed to test students' skills in analyzing and solving problems involving 3-dimensional polyhedra. A passive understanding of this material is more than what is needed for the present course.

Assignments for Sections IV.2 – IV.4

Working the exercises listed below is **strongly recommended.**

1. (*a*) Derive the formula for finding the perpendicular projection of a point $x \in \mathbb{R}^n$ onto the hyperplane H defined by the equation $\sum_i \ x_i = 1$

(*b*) If A is the simplex in $\mathbb{R}^n$ with vertices $\mathbf{0}, \mathbf{e}_1, \ \cdots \ , \mathbf{e}_n$ and $\mathbf{a} \in A$, prove that the unique line $L(\mathbf{a})$ which passes through a and is perpendicular to H meets the latter in a point on the face F opposite $\mathbf{0}$. [*Hint:* You can recover the normal direction to H from its defining equation, and $F = H \cap A$.]

(*c*) Let S be the union of all other maximal faces on A. Prove that $L(\mathbf{a})$ also meets S at some point. Note that the points in this exercises and the preceding one will be the same if $\mathbf{a} \in S \cap F = H \cap A$.

2. Using the preceding exercise, show that both S and F are strong deformation retracts of A. The homotopy should mover points along lines which are perpendicular to H.

For these problems, it might be extremely helpful to draw a picture in order to analyze the special case $n = 2$.

Readings for Unit V

The same conventions described above also apply here.

`green-chains.pdf`

Discussion of how 2-dimensional chains are relevant to proving general versions of Green's Theorem relating line integrals over a closed curve and double integrals over regions bounded by such a curve.

`chainboundary.pdf`

Detailed verification that $d \circ d = 0$ for simplicial chains.

`chain-contraction.pdf`

Detailed verification that a starshaped simplicial complex has the same simplicial homology as a one point complex.

`triple-exactness.pdf`

Detailed derivation for the long exact homology sequence associated to a short exact sequence of chain complexes.

`expansion.pdf`

Definition of an elementary expansion and a proof that the inclusion of a simplicial complex in an elementary expansion induces isomorphisms in homology.

`mv-example.pdf`

Application of the simplicial Mayer-Vietoris exact sequence to compute the homology of a specific polyhedral figure in 3-space.

Assignments for Unit V

Working the exercises listed below is **strongly recommended.**

The following exercises are taken from `exercises03-2012.pdf`:

1–2, 3a, 4ab,5c, 7, 11

Here are some additional exercises which are modified versions of problems from Hatcher. In each case some of the extra structure (for example, a linear ordering of the vertices) may be omitted.

1. A three stranded parachute complex is given by the subcomplex of a simplex with vertices a, b, c, d consisting of all the edges together with the 2-simplex abc. Compute the homology of this complex.

2. Let $(P, \mathbf{K})$ be a connected simplicial complex, and let $\mathbf{L}$ be the subcomplex consisting of m vertices in $\mathbf{K}$. Compute the relative homology groups $H_q(\mathbf{K}, \mathbf{L})$.

Reading assignments from solutions to exercises

The solutions to these exercises in `solutions03.pdf` and `solutions03a.pdf` should be read and understood at the passive level. Here is the difference between passive and active understanding:

> A passive understanding means that one can follow the reasoning presented in a written proof fairly well.
>
> An active understanding means that one knows the argument well enough to explain it correctly — or nearly so — to someone else (for example, on a quiz or examination).

Here are the exercises are taken from `exercises03-2012.pdf`:

8, 9, 10, 13, 14

Readings for Unit VI

The same conventions described above also apply here.

`existence+uniqueness.pdf`

Assignments for Unit VI

Working the exercises listed below is **strongly recommended.**

The following exercises are taken from `exercises04-2012.pdf`; note that there are two problems numbered "3":

0, 1, 2, 3(*second*), 4, 5, 6, 11

The following exercise is taken from `mv-exercises.pdf`:

3, 4

Here is one additional exercise which is a modified version of a problem from Hatcher.

1. Compute the homology of $S^n \times (S^n \vee S^n)$.

Reading assignments from solutions to exercises

The solutions to these exercises in `solutions04.pdf` and `mv-solutions.pdf` should be read and understood at the passive level as described above.

Here are the exercises are taken from `exercises04.pdf`:

3 ($first$), 4, 5, 6, 11

Here are the exercises are taken from `mv-exercises.pdf`:

1, 2