Math 205B
Winter 2019

UPDATED GENERAL INFORMATION — FEBRUARY 7, 2019

Update files from 2018

Instead of cutting and pasting, I shall comment on items from these files which are particularly important for this course up to the midterm examination.

`aabUpdate01.205B.w18.pdf` This file contains various recommended readings from files in the course directory. The most important are those involving covering spaces.

`aabUpdate03.205B.w18.pdf` This file contains recommeded exercises and readings for Unit III. The files with illustrations are definitely the most important ones at this point.

`aabUpdate05.205B.w18.pdf` There is more review material for Unit III. **Note that Section IV.1 will not be covered on the exam.** A large number of practice problems appear at the end of this file; nearly all involve material covered on the exam.

`aabUpdate06.205B.w18.pdf` This lists old examination files (with solutions) that are worth reviewing. The Winter 2018 file `exam1w18key.pdf` should be added to these lists.

`aabUpdate07.205B.w18.pdf` This corrects a misprint in file 05.

More specific comments on the midterm examination

This will cover those parts of Units I–III which are noted in the class schedule, and it will also have a problem on computing fundamental groups using the Seifert-van Kampen Theorem (so make sure you go through these problems and their solutions). There will be an example involving graphs, and you might have to do more work in sketching this example than was required in previous examinations. Several more practice problems are given below. Studying these should probably have priority over the earlier practice problems.

1. Let $U \subset \mathbb{R}^n$ be a nonempty open, (arcwise) connected subset, and let H be a subgroup of $\pi_1(U)$. Show that there is a connected covering space $p : W \to U$ such that $\mathrm{Image}\, p_* = H$.

2. Let X be an arcwise connected space such that $X = U_1 \cup U_2$, where each U_i is open and arcwise connected and $U_1 \cap U_2$ is also arcwise connected. Suppose further that $\pi_1(U_1 \cap U_2) \to \pi_1(U)$ is onto and $\pi_1(U_1 \cap U_2) \to \pi_1(U_2)$ is trivial. Prove that $\pi_1(X)$ is trivial.

3. The gas-water-electricity network has six vertices A, B, C, G, W, E such each of the first three vertices is connected to each of the last three vertices by an edge (and there are no other edges connecting pairs of vertices). Compute the fundamental group of this graph and find a maximal tree in it.

4. Let $p : E \to B$ be a covering space projection such that E is simply connected, and let $A \subset B$ be simply connected. Prove that the inclusion $j : A \to B$ lifts to a 1–1 continuous mapping $A \to E$ and that p maps the image A' of this lifting homeomorphically onto its image.

5. (a) Suppose that X is a connected graph with V vertices and E edges. Prove that $V \leq E+1$, and equality holds if and only if X is a tree.

In the setting of the preceding problem, let V_k be the number of vertices which lie on exactly k edges. Prove that $2E = \sum_k kV_k$.

6. Suppose that a connected graph X is a union of two subgraphs $X_1 \cup X_2$ such that each is a tree and their intersection is a single vertex. Prove that X is a tree.