



$$\gamma = \beta + (-\alpha)$$

$$f\gamma \approx$$

$\gamma + \alpha \approx \beta$  preserving endpts.

Covering homotopy property  $\Rightarrow$  lifts of  $f(\gamma + \alpha)$ ,  $f\beta$  have same endpts.

Use hypothesis  $\text{Im } f_* \subseteq \text{Im } p_*$  in  $\pi_1$ .  
Then  $f\gamma \approx_* p\theta$  some closed curve  $\theta$  in  $E$ .

$\theta$  lifts  $p\theta$ , so  $\underbrace{p\theta + f\alpha}_{\text{LIFTS}} + f\gamma + f\alpha$  have same endpts.

$\varphi$  lifts  $f\alpha \Rightarrow$  lift  $p\theta + f\alpha$  must be  $\theta + \varphi$  Hence  
endpt  $\widetilde{p\theta + f\alpha} = \text{endpt } \varphi = \text{endpt } f\alpha$ , so

endpt,  $\widetilde{f\alpha} = \text{endpt } \widetilde{f\gamma + f(\alpha)} = \text{endpt } \widetilde{f\beta}$ , which is what we needed.