From the previous course

1. Concept of homotopy for continuous maps from one space X to another space Y.
2. Base points and base point preserving maps.
3. Formal definitions and properties for the fundamental group of a pointed space.
4. Computation of $\pi_1(S^1, 1)$ and its far-reaching generalizations:

   A. $p: \mathbb{R} \to S^1$, $p(t) = e^{2\pi i t}$, as a special case of a covering space projection.

   B. Path Lifting Property and Covering Homotopy Property (assuming T_2 + connected + locally arcwise connected).

C. If $p: X \to Y$ is a covering space projection where X, Y are T_2, conn., loc. arc. conn., then

$$p_*: \pi_1(X, x_0) \longrightarrow \pi_1(Y, y_0)$$

is 1-1

$\boxed{\text{Q: What can we say about } \pi_1(Y, y_0)/\text{Image } p_* \text{ ??}}$

5. LIFTING THEOREM: $p: X \to Y$ as above, $g: (A, a_0) \longrightarrow (Y, y_0)$ cont. with A satisfying the conditions in 4B. Then there is a lifting $G: (A, a_0) \longrightarrow (X, x_0)$ s.t. $p \circ G = g \iff$

$$\text{Image } \pi_1(A, a_0) \to \pi_1(Y, y_0)$$

is contained in

$$\text{Image } \pi_1(X, x_0) \longrightarrow \pi_1(Y, y_0).$$

If such a lifting exists, it is unique.

CONSEQUENCE. If $p : (X, x_0) \to (Y, y_0)$ and $q : (Z, z_0) \longrightarrow (Y, y_0)$ are covering space projections (usual conditions on spaces) and the images of $\pi_1(X, x_0)$ and $\pi_1(Z, z_0)$ in $\pi_1(Y, y_0)$ are equal, then there is a unique homeomorphism $h : (X, x_0) \longrightarrow (Z, z_0)$ such that $q \circ h = p$.

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Two immediate objectives:

if Y is "nice"

(I) Realize every subgroup of $\pi_1(Y, y_0)$ by some covering space projection, and determine what one can say about the covering space projection when the subgroup is normal.

(II) Find reasonably effective ways of computing $\pi_1(Y, y_0)$ if Y is "relatively nice."