HINTS, CORRECTIONS AND SOLUTIONS TO

SELECTED EXERCISES FOR

MATHEMATICS 205B

Winter 2008

PART 5

Munkres, Section 79

A3. *Hint.* Why does X have a countable base of evenly covered open sets, and how can one use this to show that E is second countable?

A4. *Hints.* (i) Why does f lift to a map from A into E? Use this lifting to construct a cross section for the pullback covering space.

(ii) Why does it suffice to show that all points in the inverse image of the base point lie in the same arc component of $E|A$, and how can one use the surjectivity of f_* to prove this?

(iii) Explain why the composites $E|A \to E \to X$ and $E|A \to A \to X$ are identical, and conclude that the same holds for the restrictions to each connected component of $E|A$. Why is the induced map of fundamental groups trivial for the first composite and injective for the second?

Munkres, Section 80

M1(a). *Hint.* Assume all spaces are connected, let E denote the universal covering space with projection $\theta : E \to Z$, and start by showing that θ lifts to X.

Munkres, Section 81

M5. *Hint and reference.* This generalizes the examples of simple lens spaces in the commentaries. One reference for the statements in part (b) of the problem is M. Cohen, *A Course in Simple Homotopy theory* (Springer-Verlag).

Munkres, Section 84

M2. *Hint.* Compute the Euler characteristics of the spaces in question.

A1. *Hint.* The vertices are given by the vertices of the original graph structure plus one non-vertex point on each edge. How many edges of the derived structure lie in an edge of the original one?

A3. *Hint.* Imitate the earlier proofs of results showing finiteness in special cases. Local tameness implies that each point on the graph has an open neighborhood which meets only finitely many components of the complement, and for two non-vertex points on the same edge one must check that these (two) components are the same.

A4. *Hint.* Start by figuring out everything when $b - a = 1 = d - c$, and proceed by induction on the positive integers $b - a$ and $d - c$.

Munkres, Section 85

M2. *Hint.* Show that the normal subgroup generated by H also has infinite index. Why does this suffice?

H8. *Hint.* Assume that we have a connected linear graph in the sense of Munkres whose fundamental group is a given finitely generated free group. Why does it suffice to show that there are only finitely many equivalence classes of based n-sheeted connected covering spaces over X?

Explain why every n-sheeted covering space of X is equivalent to a graph whose vertices are $\mathbf{V} \times \{1, \cdots, n\}$, where $\mathbf{V}$ is the set of vertices, and whose edges contain certain pairs of vertices $(\mathbf{a}, p)$ and $(\mathbf{b}, q)$ such that $\mathbf{a}$ and $\mathbf{b}$ are vertices of some edge in X; in fact, one has edges for exactly n out of the n^2 possibilities for p and q. Show that there are only finitely many possibilites for the set of edges in the n-sheeted covering.