NAME: ______________________________

Mathematics 205B, Winter 2008, Examination 1

Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets.

#	SCORE
1	
2	
3	
4	
TOTAL	

1. [*20 points*] Explain the meanings of the Path Lifting Property and Covering Homotopy Property for the map $p(t) = \exp(2\pi \mathbf{i} t)$ from $\mathbf{R}$ to S^1, and explain how these can be used to construct a well-defined map from $\pi_1(S^1, 1)$ to $\mathbf{Z}$.

2. [*25 points*] Suppose we are given a continuous mapping $f : X \to Y$ and two continuous mappings $g : Y \to X$ and $h : Y \to X$ such that $g \circ f$ is homotopic to the identity on X and $f \circ h$ is homotopic to the identity on Y. Prove that g is homotopic to h. [*Hint:* Look at the homotopy class of $g \circ f \circ h$ in two different ways. Recall that if u is homotopic to u' as maps from A to B and v is homotopic to v' as maps from B to C, then $v \circ u$ is homotopic to $v' \circ u'$.]

3. [*30 points*] (*a*) Suppose that A is a retract of X and $x_0 \in A$. Let $j : A \to X$ be the inclusion mapping. Prove that the associated map of fundamental groups j_* is injective.

(*b*) Suppose that $x_0 \in U$, where U is open and dense in X, and give examples where the homomorphism $j_* : \pi_1(U, x_0) \to \pi_1(X, x_0)$ is (*i*) injective but not surjective, (*ii*) surjective but not injective.

4. [*25 points*] Suppose that $p : E \to B$ is a covering space projection. For each of the following statements, either give a proof or a counterexample.

(a) If B is connected, then so is E.

(b) If B is $\mathbf{T_1}$ (one point subsets are closed), then so is E. [*Hint:* What conclusions if any can be drawn about $p^{-1}[\{b\}]$ for $b \in B$?]