

Review — I

Here are some suggestions for review in connection with the midterm examination to be held on Friday, February 15, 2008.

The coverage extends through Section 60 of Munkres and the accompanying material in `commentaries.pdf` as well as the exercises listed in `math205Bexercises.pdf`. As always, some parts are more important than others. It is important to understand the basic definitions and properties of homotopic mappings, both pointed and unpointed, and the construction of the fundamental group (although there will be no questions asking for proofs that one gets a well-defined group operation from concatenation of curves). It is also important to be able to describe the basic steps in the proof that $\pi_1(S^1)$ is infinite cyclic, including the path lifting and covering homotopy properties. Similarly, the basic definitions and facts concerning covering space projections should be understood (and particularly the discussion in the posted solution to Exercise M53.6), and the ability to explain how one can realize every finitely generated abelian group as a fundamental group may be included in the examination. In a similar vein, it might be helpful to be able to give examples for which the fundamental group is not abelian, although constructions of examples will not be part of the exam. The relation between the no-retraction theorem and the Brouwer Fixed Point Theorem should also be understood well enough to explain the main ideas.

Since Sections 58–60 were covered rapidly in some parts and out of order in others, more will be said about their role in the exam. In Section 58 the most important things to know are the definitions of retracts and deformation retracts, the theorems about their associated maps of fundamental groups, the transitivity relations (a retract followed by a retract is again a retract, *etc.*), and how retracts and fundamental groups can be used to show that some pairs of topological spaces are not homeomorphic to each other. It is not necessary to know the details of elementary but somewhat messy constructions like the ones left undone in Munkres. The computations in Sections 59 and 60 were in many respects part of the course discussion for sections 53 and 54; more specialized things like information about the fundamental group of the genus two surface won't appear on the exam. Something about the proof of the main theorem in Section 59 may be on the exam. Nothing involving Sections 56 or 57 will appear on the exam.

Also, it will not be necessary to understand the solutions to Additional Exercises 55.3, 58.1–58.4 and 60.1 (or any of the other starred exercises from earlier sections). A second set of hints, *etc.* for exercises will be posted shortly.