EXERCISES FOR MATHEMATICS 205C

SPRING 2011

Addendum to File Number 00

The problem uses Exercise 00.3 to prove that $\pi_1(S^n, e_1)$ is trivial, where e_1 is the first unit vector in $\mathbb{R}^{n+1}$.

7. Prove that S^n is simply connected for all $n \geq 2$. [*Hint:* Since S^n is a deformation retract of $\mathbb{R}^{n+1} - \{\mathbf{0}\}$, it suffices to prove this for $\mathbb{R}^{n+1} - \{\mathbf{0}\}$. By Exercise 00.3 it suffices to that if γ is a closed broken line curve which is based at e_1 and has corner points with rational coordinates, then γ is homotopically trivial. As usual, let the corner points be given by $\gamma(x_i)$, and let W_i be the proper vector subspace spanned by $\gamma(x_i)$ and γ_{i-1}; this subspace is proper because $n + 1 \geq 3$. Why is there a unit vector $\mathbf{u}$ such that the span of $\mathbf{u}$ and each W_i have no nonzero vectors in common? Why does this imply that the image of γ is contained in $\mathbb{R}^{n+1} - \text{Span}(\mathbf{u})$, why is the latter homeomorphic to $(S^n - \{\mathbf{u}\})$, and why is the this space homeomorphic to $\mathbb{R}^{n+1}$? Show that the preceding observations imply that γ must be homotopically trivial.]