Mathematics 205C, Spring 2011, Assignment 1

This will be due on **Wednesday, April 20, 2011, at 11:10 A.M.** at the beginning class or by prior arrangement in my mailbox or at the front desk of Surge 202 at the same time. If you wish to use some version of TEX in writing up your answers, please feel free to do so. *You must show the work behind or reasons for your answers.*

1. Let L be a $(2n-1)$-dimensional lens space whose fundamental group is cyclic of order 60. Show that two connected covering space projections $p : M \to L$ and $q : N \to L$ hare homeomorphic domains if and only if they are equivalent as covering spaces, and determine the number of equivalence classes of covering space projections over L (there are only finitely many). [*Hint:* Look at the subgroups of a cyclic group of order 60.]

2. Suppose that the topological space X is the union of the arcwise connected open subsets U and V and that $U \cap V$ is also arcwise connected. Let $p \in U \cap V$, and assume that the canonical map from $\pi_1(U \cap V, p)$ to $\pi_1(U, p)$ is an isomorphism. Prove that the canonical map from $\pi_1(V, p)$ to $\pi_1(X, p)$ is also an isomorphism. [*Hint:* Show that the commutative diagram

$$\begin{array}{ccc}
\pi_1(U \cap V, p) & \xrightarrow[\cong]{\varphi} & \pi_1(U, p) \\
\Big\downarrow j_* & & \Big\downarrow j_*\varphi^{-1} \\
\pi_1(V, p) & \xrightarrow{=} & \pi_1(V, p)
\end{array}$$

has the Universal Mapping Property for pushouts.]