

**Mathematics 246A**  
**Algebraic Topology — I**  
**Detailed Table of Contents**  
**Fall 2010**

**Department of Mathematics**  
**University of California, Riverside**

# Detailed Table of Contents

<b>Preface</b> .....	1
Overview of the course .....	3
<b>Prerequisites</b> .....	4
Set theory .....	4
Topology .....	4
Algebra .....	5
Analysis .....	5
<b>I. Foundational and geometric background</b> .....	6
1. Categories and functors .....	6
Examples of categories .....	7
Functors .....	9
Examples of covariant functors .....	9
Contravariant functors and examples .....	10
Properties of functors .....	12
Natural transformations .....	13
Equivalences of categories .....	15
2. Barycentric coordinates and polyhedra .....	15
Affine independence and barycentric coordinates .....	16
Sets with simplicial decompositions .....	17
Decompositions of prisms .....	18
3. Subdivisions .....	21
Simple examples .....	22
Definition of subdivisions .....	22
Barycentric subdivisions .....	23
Diameters of barycentric subdivisions .....	25
4. Cones and suspensions .....	27
The constructions and their properties .....	27
A homeomorphism problem .....	29
<b>II. Homotopy and cell complexes</b> .....	31
1. Homotopic mappings .....	31
Special types of homotopy equivalences .....	32
Counting homotopy classes .....	33
Important standard notation .....	34
2. The fundamental group .....	35
3. Abstract cell complexes .....	35
Adjoining cells to a space .....	36
Cell complex structures .....	38
4. The Homotopy Extension Property .....	40

III.	<b>Simplicial homology</b> .....	43
	Some motivation from vector analysis .....	43
1.	Exact sequences and chain complexes .....	44
	Examples .....	45
	Graded objects .....	45
	Chain complexes .....	46
2.	Homology groups .....	48
3.	Homology and simplicial complexes .....	49
	Three definitions of simplicial homology groups .....	50
	Acyclic complexes .....	53
	The Königsberg Bridge Problem .....	50
4.	Comparison principles .....	54
	Application to simplicial complexes .....	56
	The Five Lemma .....	58
	The isomorphism theorem .....	58
	Application to barycentric subdivisions .....	61
5.	Chain homotopies .....	61
	An important example .....	62
IV.	<b>Singular homology</b> .....	64
1.	Definitions .....	64
	Some simple properties of homology groups .....	65
2.	Eilenberg-Steenrod properties .....	66
	The Homotopy and Excision Properties .....	67
	Excision and adjoining cells to a space .....	69
	Equivalence of singular and simplicial homology .....	70
	Homeomorphism types of spheres and Euclidean spaces .....	70
3.	Computations .....	70
	Betti numbers and torsion coefficients .....	73
	Cellular homology .....	73
	Convex linear cells .....	76
	Rational homology .....	78
	Euler characteristics and Euler's Formula .....	79
4.	Proofs of homotopy invariance and excision .....	81
	Homotopy invariance .....	81
	Barycentric subdivision of singular chains .....	82
	Small singular chains .....	83
	Application to Excision .....	84
	Mayer-Vietoris sequences .....	85
5.	Homology and the fundamental group .....	87

V.	<b>Geometric applications</b> .....	87
1.	Degree theory .....	87
	Linear algebra and degree theory .....	91
	The Fundamental Theorem of Algebra .....	92
2.	Classical theorems of Jordan and Brouwer .....	93
	Further results .....	96
3.	Simplicial approximation .....	97
4.	The Lefschetz Fixed Point Theorem .....	98
	Vector fields on $S^2$ .....	99
5.	Dimension Theory .....	100
	The basic setting .....	101
	Čech homology groups .....	103
	The basic setting .....	101
	Dimensions of products .....	108
	Counterexamples to the general question .....	108
	Further results .....	109
	Continuity in Čech homology .....	110
	Singular and Čech homology of the Polish circle .....	110
	Dimensions of nowhere dense subsets .....	113
	<b>Appendix:</b> The Flag Property .....	115
VI.	<b>Cohomology</b> .....	116
	A useful result .....	116
1.	The basic definitions .....	117
2.	A weak Universal Coefficient Theorem .....	120
	The Kronecker index .....	121
	Manipulations with dual vector spaces .....	121
3.	Examples of cup products .....	123
	Cross products .....	123
	Products of cell complexes .....	124
	Cohomology of the torus $T^n$ .....	125
	More general products with spheres .....	126
4.	Two applications .....	128
	Cell decompositions for products of spheres .....	128
	Homotopically nontrivial mappings of spheres .....	130