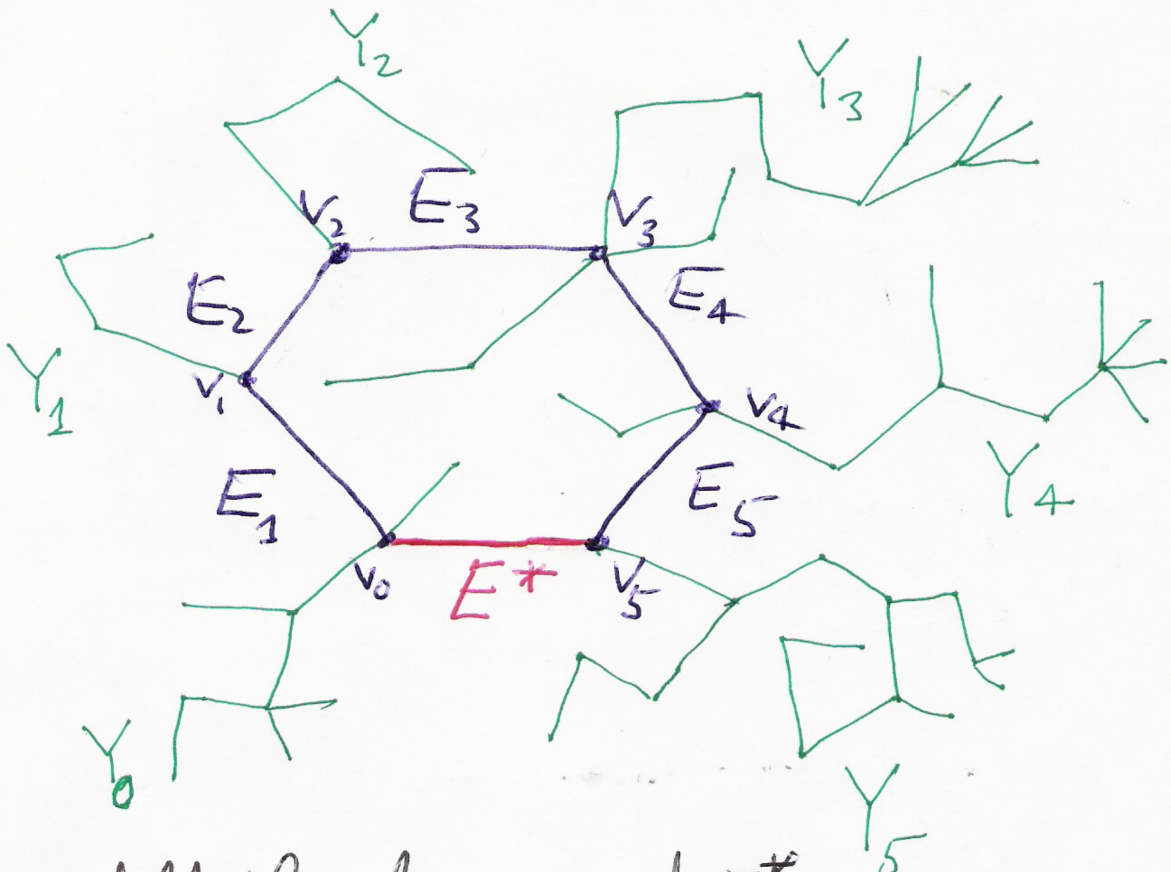
Tree-plus-one graphs



All the edges except E^* form a tree, say T. Removing the E_i's yields a finite union of pairwise disjoint trees. CLAIM Each of the v_i's lies on a different tree* and each tree contains some v_i.

* By convention, a single vertex is a tree with zero edges

If two v_i's lie in the same component of the complex Y, then there are two simple paths joining these vertices: One on the circuit $v_0 v_1 \cup \cdots \cup v_{n-1} v_n$ and one on Y (the first path avoids $E^* = v_n v_0$).

If x is a vertex in Y, (x not a v_i), then x lies in the same component of some v_i. In the tree T, there is a simple edge path joining x to v_0. Look for the first edge which has some v_i as a vertex. Then the edge path from x to v_i must be contained in the component of v_i.

π_1 of a graph

$T \subseteq X$ maximal tree

$X = T \cup E_1 \cup \dots \cup E_n$.

$W = X$ with centerpoints removed from each E_i

CLAIM: T is a deformation retract of W. — Shrink $\boxed{E_i - \text{CENTER}}$ back to the endpoints of E_i.

$U_i = W \cup$ all E_j's for $j < i$

$V_i = W \cup E_i$ So $U_i \cup V_i = U_{i+1}$

$U_{n+1} = X$.

$\boxed{\text{Seifert-van Kampen}} + \boxed{\text{Induction}} \Rightarrow$

$\pi_1(U_{i+1})$ is free on i gens.

$\Longrightarrow \pi_1(X)$ free on n gens.