Let $f : X \longrightarrow Y$ be a quotient map ( = a continuous onto map such that $Y$ has the quotient topology with respect to $f$ ). If $X$ is locally arcwise connected, then so is $Y.$

A proof is given in the following online document :

$\longrightarrow$