Math 205C
Winter 2016

UPDATED GENERAL INFORMATION — MAY 6, 2016

Review for the midterm examination

The examination will consist of mathematical problems, with no definitions or statements of theorems as such (but partial credit is possible for correct statements of relevant facts). There will be four problems, some of which are likely to touch on concepts or facts from 205A. Some problems may involve giving or working with relatively simple examples (for example, show that a given map from $\mathbf{R}^n$ to itself is, or is not, a diffeomorphism; note that such problems may require computing Jacobians). Here are a couple of other points:

One simple way of characterizing the smooth product structure on a cartesian product $M \times N$ is that maps into $M \times N$ are smooth if and only if their coordinate projections are.

The tangent space construction is a covariant functor. Also, when working with the tangent space it is usually better to use the main features of the construction rather than the construction itself.

If U is open in $\mathbf{R}^n$, V is open in $\mathbf{R}^m$, $f : U \to V$ is smooth and $x \in U$, then if $Df(x)$ has maximum rank $r = \min(m, n)$ then $Df(y)$ also has rank r for all nearby points. This is true because the matrix $Df(x)$ has an $r \times r$ submatrix whose determinant is nonzero, and by the continuity of the determinant the same condition holds for $Df(y)$ provided y is sufficiently close to x.