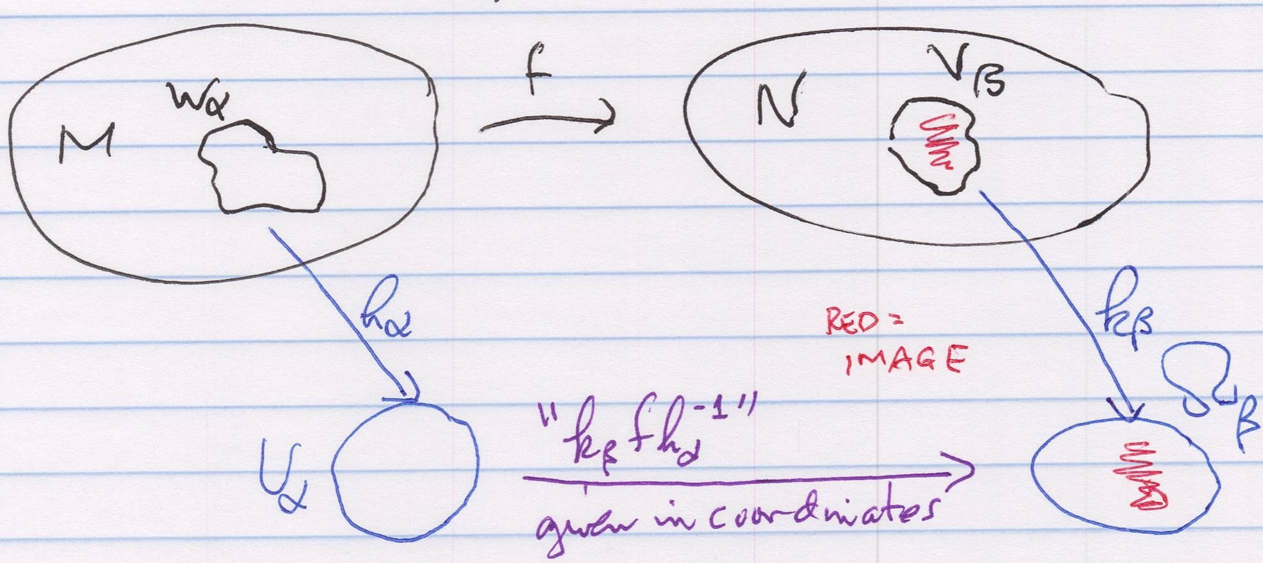
2. Smooth mappings

Let $(M, \mathcal{O}\!\mathcal{l})$ and $(N, \mathcal{B})$ be smooth manifolds. A function $f : M^m \longrightarrow N^n$ is <u>smooth</u> if

① it is continuous,

② for each $x \in M$ there are smooth charts $h_\alpha : W_\alpha \longrightarrow U_\alpha$, $k_\beta : V_\beta \longrightarrow \Omega_\beta$ such that $f[W_\alpha] \subseteq \Omega_\beta$ and "$k_\beta f h_\alpha^{-1}$" is a smooth map from U_α to V_β in the usual sense.



NOTE. By continuity one can always find smooth charts with $f[W_\alpha] \subseteq \Omega_\beta$ for each $x \in M$.

<u>Lee, Prop. 2.5</u> If f is smooth in the preceding sense, then for every h_α, k_β (smooth charts) with $f[W_\alpha] \subseteq V_\beta$ the map "$k_\beta f h_\alpha^{-1}$" is smooth.

The idea of the proof is very similar to that of Lee, Prop. 1.17(a). ■

PROPERTIES OF SMOOTH MAPS.

① Let $\mathcal{U} = \{U_\alpha\}$ be an open covering of M, let $(M, \mathcal{O})$ & $(N, \mathcal{B})$ be smooth manifolds, and let $f : M \longrightarrow N$ be continuous. Then f is smooth $\Longleftrightarrow$ for α the restrictions $f|U_\alpha$ are smooth (with the previously described smooth structure).

② The identity map on $(M, \mathcal{O})$ is smooth.

③ All constant maps are smooth.

④ If V is open in N and $f[M] \subseteq V$, then f is smooth $\Longleftrightarrow$ the corestriction $V|f$ is smooth.

⑤ If $f : (M, \mathcal{O}) \longrightarrow (N, \mathcal{B})$ is smooth and $g : (N, \mathcal{B}) \longrightarrow (P, \mathcal{C})$ is smooth, then so is $g \circ f : (M, \mathcal{O}) \longrightarrow (P, \mathcal{C})$.

<u>Def.</u> $f : (M, \mathcal{O}) \longrightarrow (N, \mathcal{B})$ is a <u>diffeomorphism</u> $\Longleftrightarrow$ f is a homeomorphism and both f and f^{-1} are smooth.

PROPERTIES OF DIFFEOMORPHISMS.

① The identity on $(M, \mathcal{O})$ is a diffeomorphism.

② The inverse of a diffeomorphism is a diffeomorphism.

③ The composite of two composable diffeomorphisms is a diffeomorphism.

④ If U & V are open in M and N, and $f : (M, \Omega) \to (N, \mathcal{B})$ is a diffeomorphism such that $f[U] = V$, then the restriction-corestriction

$$V|f|U : U \longrightarrow V$$

is also a diffeomorphism.

Proofs of the listed properties need to be added here.