Embeddings

Topological case $f : A \to X$, a cont. map of topological spaces, is an embedding if it is 1-1 and f maps A homeomorphically to $f[A]$; i.e, the corestriction $f[A]|f$ is a homeomorphism.

EXAMPLES 1. Subspace inclusions are embeddings.

nonnegative integers

2. Define $f : \mathbb{N} \to \mathbb{R}$ by $f(0) = 0$, $f(n) = \frac{1}{n}$ if $n > 0$. Then f is continuous + 1-1, but not an embedding.

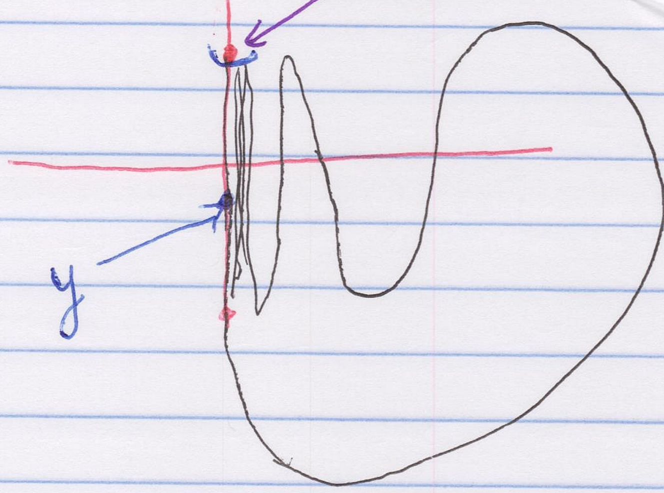
3. If we restrict the Figure 8 curve to $S^1 - \{1\} \cong (0, 2\pi)$, then we also get a continuous 1-1 map which is not an embedding [the image is compact, but the domain is not!!]

Smooth case $f : M^n \to N^n$ smooth. Then f is a smooth embedding $\Longleftrightarrow$ it is a topological embedding and a smooth immersion.

EXAMPLES 1. If M^n is compact and a smooth immersion f is 1-1, then f is automatically a smooth embedding by a standard result in point set topology.

2. The previous Example 3 is a 1-1 smooth immersion which is not a topological embedding.

POLISH CIRCLE Start with the graph of $\sin(1/x)$, then bend it back so it becomes a vertical line as follows:



This yields a 1-1 immersion which is not an embedding, for manifolds are locally arcwise connected but a point $y \in \{0\} \times [-1, 1)$ does not have a neighborhood base of arcwise connected open subsets.

*we want a set which is a continuous image of an open interval.