Note on answers for Take home assignment 1

In the last problem, one cannot conclude that, say, 1-manifolds and 2-manifolds are different because the each point x of the latter has a neighborhood base of open neighborhoods U such that $U - \{x\}$ is connected and the former has a neighborhood base of open neighborhoods V such that $V - \{x\}$ is disconnected. One can also construct a neighborhood base for points in a 2-manifold such that each of the sets $W - \{x\}$ is disconnected. For example, take $0 \in \mathbf{R}^2$ and consider the family of sets

$$N_{1/4k}(0) \quad \cup \quad N_{1/8k}(\,(1/2k, 0)\,)$$

which is a disconnected neighborhood base. The important distinguishing property of 1-manifolds is that for **ALL** sufficiently small open neighborhoods W of a point x the deleted neighborhood $W - \{x\}$ is disconnected.

A similar situation holds for 2-manifolds and 3-manifolds. In this case the distinguishing property of 2-manifolds is that for all sufficiently small connected neighborhoods W of x the set $W - \{x\}$ is not simply connected.