

**Mathematics 205C**

**Topology — III**

**Course Notes**

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**Department of Mathematics**

**University of California, Riverside**

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