

**Mathematics 246A**  
**Algebraic Topology — I**  
**Detailed Table of Contents**  
**Fall 2014**

**Department of Mathematics**  
**University of California, Riverside**

# Detailed Table of Contents

<b>Preface</b> .....		1
Course references.....		2
Overview of the course.....		4
<b>I. Further properties of simplicial complexes</b> .....		<b>6</b>
0. Review.....		6
1. Ordered simplicial chains.....		9
Acyclic complexes.....		11
Extension to pairs.....		12
The isomorphism theorem.....		13
2. Subdivisions.....		16
Explicit simple examples.....		16
Formal definition of subdivisions.....		16
Barycentric subdivisions.....		17
Diameters of barycentric subdivisions.....		20
Homology and barycentric subdivisions.....		21
3. Abstract cell complexes.....		23
Adjoining cells to a space.....		26
Cell complex structures.....		27
Cellular homology.....		29
Convex linear cells.....		31
4. The Homotopy Extension Property.....		31
5. Chain homotopies.....		34
An important example.....		35
6. Cones and suspensions.....		36
The constructions and their properties.....		36
Homological and homotopic properties of cones and suspensions.....		38
<b>II. Construction and uniqueness of singular homology</b> .....		<b>39</b>
1. Basic definitions and properties.....		40
The simplest normalization properties of homology groups.....		41
The compact supports property.....		42
2. Exactness and homotopy invariance.....		43
The exact sequence of a pair.....		43
Homotopy invariance.....		44
3. Excision and Mayer-Vietoris sequences.....		46
Barycentric subdivision and small singular chains.....		47
Small singular chains.....		28
Application to Excision.....		49
Mayer-Vietoris sequences.....		49
4. Equivalence of simplicial and singular homology.....		52
5. Polyhedral generation, direct limits and uniqueness.....		52
Polyhedral generation.....		53
Directed systems and direct limits.....		55
An isomorphism theorem for singular homology theories.....		56

III.	<b>Additional geometric applications</b> .....	58
1.	Homology and the fundamental group .....	58
2.	Degree theory .....	61
	Linear algebra and degree theory .....	63
	The Fundamental Theorem of Algebra .....	65
	Local degree theory .....	66
3.	Simplicial approximation .....	66
4.	The Lefschetz Fixed Point Theorem .....	67
	The Euler characteristic .....	67
	The Lefschetz number .....	48
	Vector fields on $S^2$ .....	69
5.	Dimension theory .....	71
	The basic setting .....	72
	Čech homology groups .....	74
	Inverse systems and inverse limits .....	74
	Definition and properties of Čech homology .....	75
	Dimensions of products .....	78
	Counterexamples to the general question .....	79
	Further results .....	80
	Continuity in Čech homology .....	80
	Singular and Čech homology of the Polish circle .....	81
	Dimensions of nowhere dense subsets .....	83
	Appendix: The Flag Property .....	86
6.	Homology and line integrals .....	87
IV.	<b>Singular cohomology</b> .....	88
	A useful result .....	88
1.	The basic definitions .....	89
	Cup products .....	90
	Relative cup products .....	91
	Simplicial cohomology .....	92
	Examples of cochains .....	93
2.	A weak Universal Coefficient Theorem .....	95
	The Kronecker Index .....	96
	Manipulations with dual vector spaces .....	96
3.	Künneth formulas .....	98
	Algebraic cross products .....	98
	Topological cross products .....	100
	The Topological Künneth Theorem .....	101
	Products in relative homology groups .....	104
	Cap products .....	106
4.	Grade-commutativity and examples .....	107
	Coalgebraic structures on simplicial chain complexes .....	108
	Algebraic and topological twist maps .....	109
	Some examples .....	110

5.	Two applications . . . . .	112
	Coefficient homomorphisms in singular homology and cohomology . . .	112
	Cell decompositions for products of spheres . . . . .	113
	Homotopically nontrivial mappings of spheres . . . . .	115
6.	Open disk coverings of manifolds . . . . .	116
	Lusternik-Schnirelmann category and cup products . . . . .	117
	References for further information . . . . .	118
7.	Real and complex projective spaces . . . . .	118
V.	<b>Cohomology and differential forms</b> . . . . .	119
0.	Review of differential forms . . . . .	120
	Covariant tensors and differential forms . . . . .	120
	Operations on differential forms . . . . .	122
	Relation to classical vector analysis . . . . .	124
1.	Smooth singular chains . . . . .	125
	Functoriality properties . . . . .	127
	Comparison principles . . . . .	128
	Smooth singular cochains . . . . .	130
2.	Generalized Stokes' Formula . . . . .	131
	Integration over smooth singular chains . . . . .	131
3.	Definition and properties of de Rham cohomology . . . . .	133
	The Mayer-Vietoris sequence . . . . .	136
4.	De Rham's Theorem . . . . .	137
	Extension to arbitrary open sets . . . . .	137
	Some examples . . . . .	139
	Generalization to arbitrary smooth manifolds . . . . .	140
5.	Multiplicative properties of de Rham cohomology . . . . .	141
6.	Path independence of line integrals . . . . .	144
	Restatements using differential forms . . . . .	146
	Some classical applications . . . . .	148