page 32, lines 18–20

The map $r(x,t)$ defined in earlier lines on this page sends $D^k \times [0,1]$ into $(D^k \times \{0\}) \cup (S^{k-1} \times [0,1])$ and is the identity on this subspace.

Verification: There are two cases because the formula defining $r(x,t)$ depends on whether $2|x|+t$ is $\leq$ or ≥ 2.

Case 1. $2|x|+t \geq 2$, so $r(x,t) = \frac{1}{|x|}(x, 2|x|+t-2)$

In this case we claim that $r(x,t) \in S^{n-1} \times [0,1]$ and the restriction of r to the latter is the identity. Let's start with the second part:

If $|x| = 1$ then $r(x,t)$ reduces to $\frac{1}{1}(x, 2+t-2=1) = (x,t)$. OK

On to the more important first part.

The first coordinate of $r(x,t)$ is $\frac{1}{|x|} \cdot x$, and hence it lies in S^{n-1}. The second coordinate is nonnegative because $2|x|+t \geq 2$ in this case, and $|x|, t \leq 1$ imply

$$2|x|+t-2 \leq 2+1-1 \leq 1. \blacksquare$$

Case 2 Now $2|x|+t \leq 2$ and $r(x,t) = \frac{2}{2-t}(x,0)$. Note that $t \leq 1$ implies the denominator $2-t \geq 1 > 0$. If $t=0$ this reduces to $(x,0)$, so the restriction to D^n is the identity. Since the first coordinate of second $r(x,t)$ is zero, it is only necessary to prove that $\frac{2}{2-t} x \in D^n$. We can rewrite the Case 2 $\leq$ as $2|x| \leq 2-t$ and from this we get

$$\frac{2}{2-t}|x| \leq 1$$, which is what we want. $\blacksquare$

page 34, line -8

Suppose we have chain maps $f, g : A \to B$ and $h, k : B \to C$ such that $D : f \simeq g$ and $E : h \simeq k$ are chain homotopies. Then $h \circ D + E \circ g$ is a chain homotopy $h \circ f \simeq k \circ g$.

Details $d(hD + Eg) + (hD + Eg)d =$

$dhD + dEg + hDd + Egd \stackrel{\text{chain maps}}{=\!=}$

$hdD + dEg + hDd + Edg =$

$(hdD + hDd) + (dE + Edg) =$

$h(g - f) + (k - h)g = hg - hf + kg - hg =$

$hf - kg.$ ■

page 40, line -3

$f: X \to Y$ continuous $\Rightarrow$ the maps

$f_\#: S_n(X) \longrightarrow S_n(Y)$, sending singular n-simplices $T: \Delta_n \longrightarrow X$ to the composites $f \circ T: \Delta_n \longrightarrow Y$, are chain maps.

Details Need only check $d f_\# = f_\# d$ on a singular simplex since these generate. But

$$d f_\# T = \sum (-1)^i (f \circ T) | \partial_i \Delta_n \underset{\text{of composition}}{\overset{\text{associativity}}{=\!=}}$$

$$\sum (-1)^i f \circ (T | \partial_i \Delta_n) = f_\# \left(\sum (-1)^i (T | \partial_i \Delta_n) \right) =$$

$$f_\# d T \blacksquare$$

page 48, line 18

If $\beta : S_*(X) \longrightarrow S_*(X)$ is barycentric subdivision and $L : \beta \simeq 1$ is a chain homotopy, then $(1 + \ldots + \beta^{r-1}) \circ L$ is a chain homotopy $\beta^r \simeq 1$.

Details

$$d(1 + \ldots + \beta^{r-1})L + (1 + \ldots + \beta^{r-1})Ld \overset{\text{each } \beta^j \text{ is a}}{\underset{\text{chain map}}{=\!=}}$$

$$\Big((1 + \ldots + \beta^{r-1}) \circ dL\Big) + \Big((1 + \ldots + \beta^{r-1}) \circ Ld\Big) =$$

$$(1 + \ldots + \beta^{r-1}) \circ (\beta - 1) = \beta^r - 1. \blacksquare$$

page 61, line -11

If $K \subseteq G$ has finite index in G, let $\overline{G} = G/[G,G]$ and let $\overline{K}$ = image K in $\overline{G}$. Then $\overline{K}$ has finite index in $\overline{G}$.

Details By construction, $\overline{K}$ is $K \cdot [G,G]/[G,G]$, so if G is the union of the cosets $a_1 K, \ldots, a_m K$ then $\overline{G}$ is the union of cosets $a_j K \cdot [G,G]/[G,G]$. ■