page 92, lines 10-11

Proof of Proposition 3.

NOTATION. If $e_0, \dots, e_r$ are the vertices of Δ_r, then $\text{Front}_p(\Delta_r)$ = simplex $e_0 \dots e_p$ and $\text{Back}_q(\Delta_r)$ = simplex $e_{r-q} \dots e_q$ (OK for all $r \geq p, q$).

(i) Naturality w.r.t. cont maps $h : (X,A) \to (Y,B)$.

Let $h : (X,A) \to (Y,B)$ be cont, let $f \in S^p(Y,B)$ and $g \in S^q(Y,B)$, and let $T : \Delta_{p+q} \to X$. Then

$$[h^{\#}(f \cup g)](T) = f \cup g\,(h \circ T) \text{ by def.,}$$

and the latter is $f(\text{Front}_p(hT)) \cdot g(\text{Back}_q(hT)) =$
$h^{\#}f(\text{Front}_p(T)) \cdot h^{\#}g(\text{Back}_q(T)) = [h^{\#}f \cup h^{\#}g](T)$.

Hence the functions $h^{\#}(f \cup g)$ and $h^{\#}f \cup h^{\#}g$ are equal (since we showed it for free generators).

This cochain lies in $S^{p+q}(X,A)$ because it vanishes if $T : \Delta_{p+q} \to A$.

$T : \Delta_{p+q} \to X$ as before

(ii) Bilinearity. $[(f_1+f_2) \cup g](T) =$

$[f_1+f_2](\mathrm{Front}_p(T)) \cdot g(\mathrm{Back}_q(T)) =$

$f_1(\mathrm{Front}_q(T)) \cdot g(\mathrm{Back}_q(T)) + f_2(\mathrm{Front}_q(T)) \cdot g(\mathrm{Back}_q(T))$

$= [(f_1 \cup g) + (f_2 \cup g)](T)$, so the cochains are equal. Similarly to prove $f \cup (g_1+g_2) = (f \cup g_1) + (f \cup g_2)$.

(iii) Associativity. T as above $\Longrightarrow$

$[(f \cup g) \cup h](T)$ and $[f \cup (g \cup h)](T)$ both

equal $f(T|e_0 \dots e_p)) \cdot g(T|e_p \dots e_{p+q}) \cdot h(T|e_{p+q} \dots e_{p+q+r})$.

(iv) Augmentation is a unit $T : \Delta_p \to X \Longrightarrow$

$$\left.\begin{aligned} f \cup \varepsilon(T) &= f(T) \cdot \underbrace{\varepsilon(T|\{e_p\})}_{=1} = f(T) \\ \varepsilon \cup f(T) &= \underbrace{\varepsilon(T|\{e_0\})}_{=1} \cdot f(T) = f(T) \end{aligned}\right\} \text{all } T.$$