page 104, line 5

Verification that the Alexander-Whitney map (on line 4) is a chain map.

As in many other instances, we start with the universal example where $T : \Delta_n \to \Delta_n \times \Delta_n$ is the diagonal map, and we write this in the form

$$(\underset{\substack{\uparrow \\ \text{1st coord}}}{x_0} \cdots x_n ; \underset{\substack{\uparrow \\ \text{2nd coord.}}}{y_0 \cdots} y_n) .$$

Then

$$d_n \psi(x_0 \cdots x_n ; y_0 \cdots y_n) = d \sum_{p=0}^{n} x_0 \cdots x_p \otimes y_p \cdots y_n =$$

$$\sum_{p=0}^{n} d(x_0 \cdots x_p \otimes y_p \cdots y_n) =$$

$$\sum_{p=0}^{n} d(x_0 \cdots x_p) \otimes y_p \cdots y_n + (-1)^p \sum_{p=0}^{n} x_0 \cdots x_p \otimes d(y_p \cdots y_n)$$

$$= \sum_{p=0}^{n} \sum_{i=0}^{p} (-1)^i x_0 \cdots \overset{\text{OMIT}}{x_i} \cdots x_p \otimes y_p \cdots y_n +$$

$$\sum_{p=0}^{n} \sum_{i=p}^{n} x_0 \cdots x_p \otimes y_p \cdots \overset{\text{OMIT}}{y_i} \cdots y_n \cdot (-1)^i$$

page 104, line 5, continued

On the other hand,

$$\psi d_n(x_0 \cdots x_n; y_0 \cdots y_n) = \psi \sum_{j=0}^{n} (-1)^j (x_0 \cdots \widehat{x_j} \cdots x_n; y_0 \cdots \widehat{y_j} \cdots y_n)$$

$$= \sum_{j=0}^{n} (-1)^j \Big[\sum_{p \le j} x_0 \cdots x_p \otimes y_p \cdots \overset{\text{OMIT}}{\widehat{y_j}} \cdots y_n +$$

$$\sum_{p \ge j} x_0 \cdots \overset{\text{OMIT}}{\widehat{x_j}} \cdots x_p \otimes y_p \cdots y_n \Big]. \quad \mathrm{I}_b$$

we subtract the second expression from the first, we are left with

$$\sum_{p=0}^{n} (-1)^p x_0 \cdots \overset{\text{OMIT}}{\widehat{x_p}} \otimes y_p \cdots y_n +$$

$$\sum_{p=0}^{n} (-1)^{p-1} x_0 \cdots x_{p-1} \otimes \overset{\text{OMIT}}{\widehat{y_{p-1}}}\, y_p \cdots y_n$$

which is zero. Hence $\psi d_n = d_n \psi$ on $(x_0 \cdots x_n; y_0 \cdots y_n)$.

page 104, line 5, continued

General case $T : \Delta_n \longrightarrow X \times Y$, $T = (T_X, T_Y)$

Note that $T_\# \Psi = \Psi T_\#$ by construction, for both evaluated at $\overbrace{(x_0 \cdots x_n; y_0 \cdots y_n)}^{= U}$ yield

$$\sum_{p=0}^{n} \mathrm{Front}_p(T_X) \otimes \mathrm{Back}_{n-p}(T_Y).$$

Then $\Psi d T = \Psi d T_\#(U) = \Psi T_\# d(U) =$

$T_\# \Psi d(U) \underset{\text{PREV}}{=} T_\# d \Psi(U) = d T_\# \Psi(U) =$

$d \Psi T_\#(U) = d \Psi(T)$.

page 104, line 16

$$d\varphi(F_X \otimes B_Y) = d(F_X \times B_Y)_\# \varphi(id_p \otimes id_{n-p}) =$$

$$(F_X \times B_Y)_\# \varphi d(id_p \otimes id_{n-p}) \overset{\text{check}}{\underset{\text{this!}}{=\!=}}$$

$$\varphi \circ (F_X \times B_Y)_\# d(id_p \otimes id_{n-p}) =$$

$$\varphi d(F_X \times B_Y)_\# (id_p \otimes id_{n-p}) = \varphi d(F_X \otimes B_Y)$$