page 107, lines 20-21

Relative cross products for closed subsets.

$\underset{\text{closed}}{A} \subseteq \underset{\text{open}}{U} \subseteq X$, $\underset{\text{closed}}{B} \subseteq \underset{\text{open}}{V} \subseteq Y$ s.t.

A is a def. retract of U & B is a def. retract of B.

Then we have

$$\begin{tikzcd}
H^*(X,A) \otimes H^*(Y,B) \arrow[drr, dashed] & & H^*(X,U) \otimes H^*(Y,B) \arrow[ll, "\cong"'] \arrow[d, "\times"', "\text{prod}"] \\
& & H^*(X\times Y, U\times Y \cup X\times V) \arrow[d, "(\text{inclusion})^*"] \\
& & H^*(X\times Y, A\times Y \cup X\times B)
\end{tikzcd}$$

Note If (K, L) is a simplicial complex pair with underlying space pair $(|K|, |L|)$, then $|L|$ is a deformation retract of an open neighborhood W in $|K|$.

page 108, lines 12-13

WORK ON THE CHAIN/COCHAIN LEVEL (see notes)

Proof of Proposition 14.

(i) Let $c \in S_q(X)$ be a chain. Then

$$\varepsilon_X \cap c = \underbrace{\varepsilon_X(T|\{e_0\})}_{=1} \cdot \underbrace{\mathrm{Back}_q(c)}_{=c} = c$$

and $c \cap \varepsilon_X = \mathrm{Front}_q(c) \cdot \varepsilon_X(T|\{e_q\}) =$ c likewise.

[$\mathrm{Back}_q = \mathrm{id}$ on q-simplices!]

(ii) Check that both cochains $(f \cup g) \cap c$ and $f \cap (g \cap c)$ are equal to $f(T|e_0 \ldots e_q) \cdot g(T|e_q \ldots e_p) \cdot \mathrm{Back}_r\, c$

where $r = n - p - q$.

(iii) Let $g \in S^p(Y)$ and $c \in S_n(X)$. Then

$g \cap f_\# c = g(\mathrm{Front}_p(f_\# c)) \cdot \mathrm{Back}_{n-p}(f_\# c) =$

$g(f_\# \mathrm{Front}_p\, c) \cdot f_\# \mathrm{Back}_{n-p}(c) =$

$f^\# g(\mathrm{Front}_p\, c) \cdot f_\#(\mathrm{Back}_{n-p}(c)) \; \dfrac{f_\# \text{ is a}}{\text{module hom}}$

page 108, lines 12–13 continued

$$f_\#\left(f_g^\#(\mathrm{Front}_p c)\cdot \mathrm{Back}_{n-p}(c)\right) =$$
$$f_\#\left(f_g^\# \cap c\right).$$

By the discussion preceding the statement of Prop 14, the corresponding identities in homology and cohomology follow directly from these.