page 109, lines -5 to -4

Proof that

(1) the Alexander Whitney map $S_*(X) \longrightarrow S_*(X)\otimes S_*(X)$

is coassociative,

(2) the Alexander-Whitney map is functorial with respect to cont. maps $f: X \longrightarrow Y$.

---

(1) Both $(\Psi\otimes id)\circ\Psi$ and $(id\otimes\Psi)\circ\Psi$ send

$$T:\Delta_m \longrightarrow X \text{ to } \sum_{0\le s\le t\le m} 2(T|e_0\ldots e_s)\otimes(T|e_s\ldots e_t)\otimes(T|e_t\ldots e_m).$$

(2) The goal is to verify that the diagram

$$\begin{tikzcd} S_*(X) \arrow[r, "\Psi_X"] \arrow[d, "f_*"'] & S_*(X)\otimes S_*(X) \arrow[d, "f_*\otimes f_*"] \\ S_*(Y) \arrow[r, "\Psi_Y"] & S_*(Y)\otimes S_*(Y) \end{tikzcd}$$

commutes.

If we apply both/either of these composites to $T:\Delta_m\to X$, the result is

$$\sum \mathrm{Front}_p(f\circ T)\otimes \mathrm{Back}_{m-p}(f\circ T).$$

page 110, lines 19-20

Verification that $\tau : A_* \otimes B_* \to B_* \otimes A_*$ is a chain map.

It suffices to show $d\tau(a\otimes b) = \tau d(a\otimes b)$ for $a \in A_p$, $b \in B_q$.

$d\tau(a\otimes b) = d[(-1)^{pq} b\otimes a] =$

$(-1)^{pq}[db\otimes a + (-1)^{p} b\otimes da] =$

$(-1)^{pq}[db\otimes a + (-1)^{q\,(p+1)q} b\otimes da$. Also,

$\tau d(a\otimes b) = \tau(da\otimes b + (-1)^p a\otimes db) =$

$(-1)^{(p-1)q} b\otimes da + (-1)^{(p-1)q}(-1)^p da\otimes b =$

★ $\updownarrow$

$(-1)^{(p+1)q} b\otimes da + (-1)^{pq} da\otimes b$,

so $d\tau(a\otimes b) = \tau d(a\otimes b)$.

★: $(-1)^{(p-1)q} = (-1)^{(p-1)q}\cdot \underbrace{(-1)^{2q}}_{=1} = (-1)^{(p+1)q}$

page 110, line 10

Singular augmentations are co-units

(1) $S_*(X) \xrightarrow{\Psi} S_*(X) \otimes S_*(X) \longrightarrow \mathbb{D} \otimes S_*(X) \cong S_*(X)$.

Evaluate on $T: \Delta_n \to X$:

$$T \longmapsto \sum_p \mathrm{Front}_p(T) \otimes \mathrm{Back}_{n-p}(T) \longmapsto \varepsilon(\mathrm{Front}_0(T)) \otimes T$$

$$\approx T$$

$\Rightarrow$ composite is the identity.

(2) $S_*(X) \xrightarrow{\Psi} S_*(X) \otimes S_*(X) \longrightarrow S_*(X) \otimes \mathbb{D} \cong S_*(X)$

Evaluate on $T: \Delta_n \to X$:

$$T \longmapsto \sum_p \mathrm{Front}_p(T) \otimes \mathrm{Back}_{n-p}(T) \longmapsto T \otimes \varepsilon(\mathrm{Back}_0(T))$$

$$\approx T$$

$\Rightarrow$ composite is the identity.

page 112, lines 7–8

This follows immediately from Theorem IV. 3.10 (p. 103).

page 112, line −1

Computation of $\left(\sum_{k=1}^{r} u_k\right)^r$.

One can use the multinomial theorem to expand this because the u_k's commute with each other. Furthermore, $u_k^2 = 0 \Rightarrow$ the only nonzero terms have the form $u_1 \cdots u_r$, and the multinomial coefficient is $\dfrac{r!}{1!\,1!\cdots 1!} = r!$

so that $\left(\sum_{k=1}^{r} u_k\right)^r = r! \prod_{k=1}^{r} u_k$.