page 139, lines 5-7

The decomposition $U = \cup K_m$, where

(i) $U \subseteq \mathbb{R}^n$ is open

(ii) K_m is compact and $K_m \subseteq \operatorname{Int} K_{m+1}$

is actually a special case of the following purely point-set-theoretic result.

Proposition Let X be locally compact T_2 and second countable. Then $X = \cup K_m$, where K_m is compact and $K_m \subseteq \operatorname{Int} K_{m+1}$.

Proof ① Claim X has a countable base of open sets U_k such that $\overline{U_k}$ is compact.

Let $\mathfrak{V} = \{V_p\}_{p \geq 0}$ be a base, and let $x \in X$, so $x \in V_{p_0}$ some p_0. Then there is an open set W s.t. $x \in W \subseteq \overline{W} \subseteq V_{p_0}$ and $\overline{W}$ is compact. Since $\mathfrak{V}$ is a base for X, we know that

$W = \bigcup_{j \in J} V_j$ for some $J \subseteq \mathbb{N}$, and each $\overline{V_j}$ is compact since $\overline{V_j} \subseteq \overline{W}$ and the latter is compact. Hence the set $\mathcal{V}'$ of all $V_q \in \mathcal{V}$ with $\overline{V_q}$ compact is also a base for X.

(2) By the preceding, there is a countable base $\mathcal{U} = \{U_k\}_{k \geq 0}$ with compact closures. Pick $x_0 \in X$, and let $x_0 \in U_{k_0}$, and set $K_0 = \overline{U_{k_0} \cup U_0}$. Given K_m, construct K_{m+1} as follows: One can find a finite collection of open sets $\{U_j\}_{j \in J \subseteq \mathbb{N}}$ in $\mathcal{U}$ such that $K_m \subseteq \bigcup_{j \in J} U_j$. Let K_{m+1} be the closure of $\left(\bigcup_{j \in J} U_j\right) \cup U_{m+1}$. Clearly $K_m \subseteq \operatorname{Int} K_{m+1}$, K_{m+1} is also compact, and $X = \bigcup_{m \in \mathbb{N}} U_m \subseteq \bigcup K_m \subseteq X$ shows $X = \bigcup_m K_m$.