page 32, lines ~~18 20~~ 16-18

The map $r(x,t)$ defined in earlier lines on this page sends $D^k \times [0,1]$ into $(D^k \times \{0\}) \cup (S^{k-1} \times [0,1])$ and is the identity on this subspace.

Verification: There are two cases because the formula defining $r(x,t)$ depends on whether $2|x| + t$ is $\leq$ or ≥ 2.

Case 1. $2|x|+t \geq 2$, so $r(x,t) = \frac{1}{|x|}(x, 2|x|+t-2)$

In this case we claim that $r(x,t) \in S^{n-1} \times [0,1]$ and the restriction of r to the latter is the identity. Let's start with the second part:

If $|x| = 1$ then $r(x,t)$ reduces to
$\frac{1}{1}(x, 2+t-2=1) = (x,t)$. OK

On to the more important first part.
The first coordinate of $r(x,t)$ is $\frac{1}{|x|} \cdot x$, and hence it lies in S^{n-1}. The second coordinate

p. 32, lines 16-18 continued

is nonnegative because $2|x| + t \geq 2$ in this case, and $|x|, t \leq 1$ imply

$$2|x| + t - 2 \leq 2 + 1 - 1 \leq 1. \blacksquare$$

<u>Case 2</u> Now $2|x| + t \leq 2$ and $r(x,t) = \frac{2}{2-t}(x, 0)$. Note that $t \leq 1$ implies the denominator $2 - t \geq 1 > 0$. If $t = 0$ this reduces to $(x, 0)$, so the restriction to D^n is the identity. Since the first coordinate of second $r(x,t)$ is zero, it is only necessary to prove that $\frac{2}{2-t} x \in D^n$. We can rewrite the Case 2 $\leq$ as $2|x| \leq 2 - t$ and from this we get

$$\frac{2}{2-t}|x| \leq 1$$, which is what we want. $\blacksquare$

page 34, line -8

Suppose we have chain maps $f, g : A \to B$ and $h, k : B \to C$ such that $D : f \simeq g$ and $E : h \simeq k$ are chain homotopies. Then $h \circ D + E \circ g$ is a chain homotopy $h \circ f \simeq k \circ g$.

Details $d(hD + Eg) + (hD + Eg)d =$

$$dhD + dEg + hDd + Egd \overset{\text{chain}}{\underset{\text{maps}}{=\!=}}$$

$$hdD + dEg + hDd + Edg =$$

$$(hdD + hDd) + (dE + Edg) =$$

$$h(g - f) + (k - h)g = hg - hf + kg - hg =$$

$$hf - kg. \blacksquare$$