

page 40, line - 3

$f: X \rightarrow Y$  continuous  $\Rightarrow$  the maps

$f_{\#}: S_n(X) \rightarrow S_n(Y)$ , sending singular  
 $n$ -simplices  $T: \Delta_n \rightarrow X$  to the composites  
 $f \circ T: \Delta_n \rightarrow Y$ , are chain maps.

Details Need only check  $d f_{\#} = f_{\#} d$  on a singular  
simplex since these generate. But

$$d f_{\#} T = \sum (-1)^i (f \circ T) | \partial_i \Delta_n \quad \frac{\text{associativity}}{d \text{ composition}}$$

$$\sum (-1)^i f \circ (T | \partial_i \Delta_n) = f_{\#} \left( \sum (-1)^i (T | \partial_i \Delta_n) \right) =$$

$$f_{\#} d T \quad \blacksquare$$

page 47, lines -9 to -8

Proof that  $\beta_q - 1_q - \sum_j (-1)^j \partial_{j\#} L_q$  is a cycle.

Details: By the formula at the top of p. 47 we have  $d\beta_q = \sum_k (-1)^k \partial_{k\#} \beta_{q-1}$ , so

$$d(\beta_q - 1_q) = \sum_k (-1)^k \partial_{k\#} (\beta_{q-1} - 1_{q-1})$$

and by the induction hypothesis this equals

$$\sum_k (-1)^k \partial_{k\#} (L_q + \sum_j (-1)^j \partial_{j\#} L_{q-1}) =$$

$$\sum_k (-1)^k \partial_{k\#} L_q + \sum_{k\#j} (-1)^{j+k} \partial_{k\#} \partial_{j\#} L_{q-1}$$

↑  
This is zero by the usual identity  $\sum_j (-1)^{j+k} \partial_{k\#} \partial_{j\#} = 0$

Therefore  $d_q$  sends

(Left Hand Side) - (Right Hand Side) to zero,

which is what we wanted to prove.

page 48, line ~~18~~ 20

If  $\beta: S_*(X) \rightarrow S_*(X)$  is barycentric subdivision and  $L: \beta \simeq 1$  is a chain homotopy, then  $(1 + \dots + \beta^{r-1}) \circ L$  is a chain homotopy  $\beta^r \simeq 1$ .

Details

$$d(1 + \dots + \beta^{r-1})L + (1 + \dots + \beta^{r-1})Ld \quad \underline{\underline{\text{each } \beta^j \text{ is a chain map}}}$$
$$\left( (1 + \dots + \beta^{r-1}) \circ dL \right) + \left( (1 + \dots + \beta^{r-1}) \circ Ld \right) =$$
$$(1 + \dots + \beta^{r-1}) \circ (\beta - 1) = \beta^r - 1. \blacksquare$$