

TOPICS FOR MATHEMATICS 246A, FALL 2012

Preface

Prerequisites

I. Further properties of simplicial complexes

0. Review
1. Ordered simplicial chains (Hatcher: 2.1)
2. Subdivisions (Hatcher: 2.1)
3. Abstract cell complexes (Hatcher: 2.2, Appendix)
4. Homotopy extension properties (Hatcher: 0, Appendix)
5. Chain homotopies (Hatcher: 2.1)

II. Construction and uniqueness of singular homology

1. Basic definitions and properties (Hatcher: 2.1)
2. Homotopy invariance (Hatcher: 2.1)
3. Excision and Mayer-Vietoris sequences (Hatcher: 2.1)
4. Equivalence of simplicial and singular homology (Hatcher: 2.1 – 2.3)
5. Polyhedral generation, direct limits and uniqueness (Hatcher: 2.1, 2.3, 3.F)

III. Additional geometric applications

1. Homology and the fundamental group (Hatcher: 2.A, 3.G)
2. Degree theory (Hatcher: 2.2)
3. Simplicial approximation (Hatcher: 2.C)
4. The Lefschetz Fixed Point Theorem (Hatcher: 2.C)
5. Dimension Theory (Munkres: 50)
6. Homology and line integrals (Lee: 6, 12, 14)

IV. Cohomology theory

1. The basic definitions (Hatcher: 2.2, 3.1, 3.2, 3.E)
2. Weak Universal Coefficient Theorem (Hatcher: 3.1, 3.A)
3. Künneth Formulas (Hatcher: 3.2, 3.B)
4. Grade-commutativity and examples (Hatcher: 3.2, 3.B)
5. Two applications (Hatcher: 3.2, 4.2)
6. Open disk coverings of manifolds (Hatcher: 3.2)
7. Real and complex projective spaces (Hatcher: 0, 1.2–1.3, 2.2, 2.C, 3.2)

V. Cohomology and differential forms

0. Review of differential forms (Conlon: 6.2, 6.4, 7.1–7.2, 8.1; Lee: 6, 11-12)
1. Smooth singular chains (Hatcher, §§ 2.1, 2.3; Conlon, § 8.2; Lee, Ch. 16)
2. Generalized Stokes' Formula (Conlon: 2.6, 8.1–8.2; Lee: 14)
3. Definition and properties of de Rham cohomology (Hatcher: 2.1, 2.3, 3.1; Conlon: 2.6, 8.1, 8.3–8.5; Lee: 15)
4. De Rham's Theorem (Conlon: 8.9; Lee: 15–16)
5. Multiplicative properties of de Rham cohomology (Hatcher: 3.1–3.2; Conlon: D.3; Lee, 15)

VI. Manifolds and orientations

1. Local orientations (Hatcher: 3.3)
2. Fundamental classes (Hatcher: 3.3)
3. Duality Theorems (Hatcher: 3.3)

References

Conlon, *Differentiable Manifolds, Second Edition*.

Hatcher, *Algebraic Topology*.

Lee, *Introduction to Smooth Manifolds*.

Munkres, *Topology, Second Edition*.