More rationalization proofs

Remaining details for p. 102 in algtopnotes2012.pdf.

① If $A \xrightarrow{f} B$ is 1-1, so is $A_{(0)} \xrightarrow{f_{(0)}} B_{(0)}$.

Proof. Suppose $[a, s] \in A_{(0)}$ goes to zero, so that $0 = [f(a), s]$.* Then $s \cdot f(a) = 0$ by definition. But $sf(a) = f(sa)$ and since f is 1-1 the condition $f(sa) = 0$ implies $sa = 0$, so that $0 = [a, s] \in A_{(0)}$.

$\boxed{\text{* Note that } f_{(0)}[a,s] = [f(a), s]}$

② If f as above is onto, so is $f_{(0)}$.

Proof. Let $[b, t] \in B_{(0)}$ and choose $a \in A$ so that $f(a) = b$. Then $[b, t] = [f(a), t] = f_{(0)}[a, t]$. Hence $f_{(0)}$ is onto. ■

[The other case is verified in the proof of Thm. 2 on page 102 of the cited notes.]