Solutions for `quiz01w21.pdf`

We are given a 3×3 matrix

$$P = \begin{pmatrix} -1 & A & B \\ 0 & C & D \\ 0 & 0 & -2 \end{pmatrix}$$

where A, B, C, D are single digit nonnegative integers. This matrix has three distinct eigenvalues -2, -1 and $C \geq 0$, and we are supposed to compute eigenvectors for each eigenvalue. This means we need to compute the solution spaces for the following systems of homogeneous linear equations:

$$(P+I)X = 0 \,, \qquad (P-CI)X = 0 \,, \qquad (P+2I)X = 0$$

The three square matrices in this display are given explicitly as follows:

$$P+I = \begin{pmatrix} 0 & A & B \\ 0 & C+1 & D \\ 0 & 0 & -1 \end{pmatrix} , \qquad P-CI = \begin{pmatrix} -(C+1) & A & B \\ 0 & 0 & D \\ 0 & 0 & -(C+2) \end{pmatrix} ,$$

$$P+2I = \begin{pmatrix} 1 & A & B \\ 0 & (C+2) & D \\ 0 & 0 & 0 \end{pmatrix}$$

The resulting system for the eigenvalue -1 is $-z = 0$, $(C+1)y + Dz = 0$ and $Ay + Bz = 0$. The solutions to this system are given by $y = z = 0$ and x arbitrary. This means that the solution space for the system in this case is generated by $x = 1$ and $y = z = 0$.

The resulting system for the eigenvalue C is $(C+2)z = Dz = 0$ and $Ay - (C+2)x + Bz = 0$. In this case the solutions are all scalar multiples of the triple $z = 0$, $y = (C+2)$ and $x = A$.

Finally, the resulting system for the eigenvalue -2 is $(C+2)y + Dz = 0$ and $x + Ay + Bz = 0$. The solutions for the first equation are all scalar multiples of the triple $z = C+2$, $y = -D$ and x arbitrary. Therefore the simultaneous solutions for both equations re all scalar multiples of the triple $z = C+2$, $y = -D$ and $x = AD - B(C+2)$.■

Note. Since $C \geq 0$ it follows that both $C+1$ and $C+2$ are positive and hence nonzero.