NAME: ______________________________

Mathematics 205B, Winter 2019, Examination 2

Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets.

Assume all spaces are Hausdorff and locally arcwise connected, and assume the existence of a singular homology theory with the properties specified in the course.

#	SCORE
1	
2	
3	
4	
TOTAL	

1. [*25 points*] Given two chain complexes (A_*, d^A) and (B_*, d^B), explain why the direct sum $(A_* \oplus B_*, d^A \oplus d^B)$ is also a chain complex and for each integer q we have an isomorphism $H_q(A \oplus B) \cong H_q(A) \oplus H_q(B)$.

2. [*25 points*] Suppose that Q is a solid square in $\mathbb{R}^2$ with vertices A, B, C, D, and take the simplicial decomposition of Q whose vertices are the latter plus the center point E; assume the ordering of the vertices is the usual alphabetical order. Find a simplicial 2-chain for the associated simplicial complex such that the boundary chain is a linear combination of AB, BC, CD, AD such that the coefficient of each edge is either $+1$ or -1.

3. [*25 points*] Find a tree which is not homeomorphic to one of the examples H, X, Y, Z, where in each case the font is identical to the one which is used for the letters. Give valid mathematical reasons for your answer.

4. [*25 points*] Prove that no two of the spaces $S^1 \times \mathbb{R}^2$, $S^2 \times \mathbb{R}^2$ and $S^2 \times \mathbb{R}$ are homeomorphic to each other. [*Hint:* The set $S^{n-1} \times \mathbb{R}$ is homeomorphic to the open subset $\mathbb{R}^n - \{\mathbf{0}\}$ in $\mathbb{R}^n$. What happens if we take Cartesian products with some $\mathbb{R}^k$?]

Extra page for use if needed