NAME: ______________________________

Mathematics 205B, Winter 2018, Examination 2

Work all questions, and unless indicated otherwise give reasons for your answers. The point values for individual problems are indicated in brackets.

Assume all spaces are Hausdorff and locally arcwise connected, and assume the existence of a singular homology theory with the properties specified in the course.

#	SCORE
1	
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TOTAL	

1. [*25 points*] Prove that there is no continuous 1–1 mapping $f : S^m \to S^n$ if $m > n$. [*Hint:* Let $j : S^n \to S^m$ denote the standard inclusion and consider the mapping $j \circ f : S^m \to S^m$. Why is it 1–1, and why is the image not open?]

2. [*25 points*] Let (X, a) and (Y, b) be pointed connected spaces, and let $f : (X, a) \to (Y, b)$ be continuous and basepoint preserving. Suppose that we are given two basepoint preserving covering space projections $p_1 : (X^*, a^*) \to (X, a)$ and $p_2 : (Y^*, b^*) \to (Y, b)$ such that X^* and Y^* are simply connected. Prove that there is a unique continuous mapping $F : (X^*, a^*) \to (Y^*, b^*)$ such that $p_2 \circ F = f \circ p_1$. Also, prove that if f is a homeomorphism then so is F.

3. [*30 points*] Let $\mathbf{K}$ be an n-dimensional simplicial complex (with underlying space P and a linear ordering ω of the vertices), and let $\mathbf{K}_0$ be a subcomplex of $\mathbf{K}$ which contains all the simplices of $\mathbf{K}$ except for one n-simplex. Furthermore, assume that $H_n(\mathbf{K}) \cong \mathbb{Z}$ with a generator of the form $\sum \varepsilon_j \, \sigma_j$, where σ_j runs through the n-simplices of $\mathbf{K}$ and each ε_j is ± 1.

(a) Using simplicial excision, explain why the map $H_n(\mathbf{K}) \to H_n(\mathbf{K}, \mathbf{K}_0)$ is an isomorphism.

(b) Why does (a) imply that $H_n(\mathbf{K}_0) = 0$? [*Hint:* What can we say about the group $H_{n+1}(\mathbf{K}, \mathbf{K}_0)$?]

4. [*20 points*] Let U and V be convex open subsets of $\mathbb{R}^n$ for some n. Prove that $H_q(U \cup V) = 0$ if $q \geq 1$. [*Hint:* What do we know about the intersection of two convex sets?]

5. [*25 points*] Suppose that X is a contractible space and $A \subset X$ is a subspace. Prove that $H_{q+1}(X, A)$ is isomorphic to $H_q(A)$ for all $q > 0$.

6. [*25 points*] Let X be a nonempty topological space, and let $j : X \to X \times [0,1]$ send x to $(x, 0)$. Prove that the induced homomorphism j_* in singular homology is an isomorphism in each dimension. [*Hint:* Consider the coordinate projection $p : X \times [0,1] \to X$ and the composites $p \circ j$, $j \circ p$.]

Extra page for use if needed