

More things to know for the second midterm examination:

1. The definition of exact sequence and how to check it for simple examples.
2. The definition of the simplicial chain complex associated to a simplicial complex with a linear ordering of the vertices, and how to do fairly simple computations.
3. The proof that the 0 — dimensional homology of a connected graph is infinite cyclic, and why this implies a similar result for the homology of an arbitrary connected complex.
4. Statements of basic axioms for singular homology such as the Mayer — Vietoris sequence axiom, the excision axiom, the compact supports axiom, and the normalization axioms.
5. Basic results on chain complexes and exact sequences such as the construction of induced homology mappings (along with the necessary proofs), the associated long exact homology sequence to a short exact sequenced of chain complexes and the Five Lemma.
6. Definition of local homology of a space at a point, the proof that it depends only on the topology of an arbitrarily small neighborhood of a point, and applications to graphs and Invariance of Dimension.
7. Statements of basic applications such as the Jordan — Brouwer Separation Theorem, the acyclicity result for complements of topological disks in the n — sphere, and Invariance of Domain (special emphasis here). Two good examples relating this and the previous point are the proofs that a closed n — disk and a closed m — disk are homeomorphic if and only if $m = n$, and also the corresponding result for closed half — spaces in n and m dimensions.