

Mathematics 205B, Winter 2019, Assignment 1

This will be due on **Monday, February 4, 2019, at 9:10 A.M.** at the beginning class or by prior arrangement in my mailbox or at the front desk of Surge 202 at the same time. If you wish to use some version of \TeX in writing up your answers, please feel free to do so.

You must show the work behind or reasons for your answers.

1. Let X be a space which is Hausdorff, connected and locally arcwise connected, let $f : (X, x) \rightarrow (S^1, 1)$ be a continuous mapping, and let $p : \mathbb{R} \rightarrow S^1$ be the map $p(t) = \exp(2\pi it)$. Prove that f lifts to a map $F : X \rightarrow \mathbb{R}$ if and only if f is basepoint preservingly homotopic to a constant mapping.

2. Let X be a space which is Hausdorff, connected and locally arcwise connected, let $x \in X$, and assume further that $\pi_1(X, x)$ is finite of order n . Prove that, up to equivalence, there are only finitely many connected covering spaces of X , and give an upper bound $B(n)$ for the number of equivalence classes of coverings as an explicit, elementary function of n .