

## BIBLIOGRAPHY

This is certainly not meant to be comprehensive, but it does list numerous books and papers that influenced the writing of these notes as well as references to some additional topics that were mentioned but not discussed in much detail. Throughout the notes there are also references to World Wide Web sites for further information on various points; in all cases, the mathematical content of online sites has been checked for reliability and meets the standards of quality expected for regular printed publications.

### 1. BOOKS COVERING BACKGROUND MATERIAL

**N. Altshiller Court.** *College Geometry: An Introduction to the Modern Geometry of the Triangle and the Circle* (2<sup>nd</sup> Rev. Enl. Ed.). Dover, New York, 2007.

**S. S. Epp.** *Discrete Mathematics with Applications* (Fourth Edition). Brooks-Cole, Boston, 2004.

**M. Hausner.** *A Vector Space Approach to Geometry*. Dover, New York, 1998.

**P. J. Kelly and E. G. Strauss.** *Analytic Geometry and Linear Transformations*. Scott-Foresman, Glenview, IL, 1970.

**J. E. Marsden and A. J. Tromba.** *Vector Calculus* (5<sup>th</sup> Ed.). W. H. Freeman & Co., New York, 2003.

**E. E. Moïse and F. L. Downs.** *Geometry*. Addison-Wesley, Reading, MA, 1964. [There are also numerous later versions of this text, the last of which was published in 1991, and there are standard accompanying materials such as Teachers' Editions.]

**G. Pólya.** *How to solve it*. Princeton Univ. Press, Princeton, 2004.<sup>1</sup>

**A. Pettofrezzo.** *Vectors and their applications*. Dover, New York, 2005.

**K. H. Rosen.** *Discrete Mathematics and its Applications* (Sixth Edition). McGraw-Hill, Columbus, OH, 2007.

**S. Schuster.** *Elementary Vector Geometry*. Wiley, New York, 1962.

### 2. CULTURAL AND HISTORICAL DISCUSSIONS OF PROJECTIVE GEOMETRY

**N. Altshiller Court.** *Mathematics in Fun and in Earnest*. Dover, New York, 2006.

**E. T. Bell.** *Men of Mathematics*. Touchstone Books (Simon and Schuster), New York, 1986.<sup>2</sup>

**C. B. Boyer and U. C. Merzbach.** *A History of Mathematics* (2<sup>nd</sup> Rev. Ed., with a foreword by Isaac Asimov). Wiley, New York, 1991.

**J. L. Coolidge.** *A History of Geometrical Methods*. Dover, New York, 1963.

**J. L. Coolidge.** *A History of the Conic Sections and Quadric Surfaces*. Dover, New York, 1968.

**J. L. Coolidge.** *The Mathematics of the Great Amateurs*. Dover, New York, 1963.

**R. Courant and H. Robbins.** *What is Mathematics? An Elementary Approach to Ideas and Methods* (2<sup>nd</sup> Ed.). Oxford Univ. Press, New York, 1996.<sup>3</sup>

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<sup>1</sup>This has a classic book on problem solving for half a century. GEORGE PÓLYA (1887–1985) worked in probability, analysis, number theory, geometry, combinatorics and mathematical physics.

<sup>2</sup>This book is a fascinating piece of literature, but its historical scholarship is extremely inaccurate in many places.

<sup>3</sup>RICHARD COURANT (1888–1972) is known for his work on differential equations, for several outstanding books that have remained timely over many decades, and his establishment of a prestigious mathematical institute which now carries his name. HAROLD ROBBINS (1915–2001) made several major contributions to mathematical statistics and also did research in several other areas.

**V. J. Katz.** *A History of Mathematics: An Introduction* (2<sup>nd</sup> Ed.) Addison-Wesley, Boston, MA, 1998.

**M. Kline.**<sup>4</sup> *Mathematical Thought from Ancient to Modern Times.* Oxford Univ. Press, New York, 1972.

**M. Kline.** *Mathematics in Western Culture.* Oxford Univ. Press, New York, 1964.<sup>5</sup>

**J. R. Newman (ed.).** *The World of Mathematics, Volumes 1 – 4.* Dover, New York, 2003.

### 3. SIMILAR MATERIAL TO THE NOTES AT COMPARABLE LEVELS

(The references at the end of Appendix E also fit into this category.)

**R. Artzy.** *Linear Geometry.* Addison-Wesley, Reading, MA, 1963.

**M. K. Bennett.** *Affine and Projective Geometry.* Wiley-Interscience, New York, 1995.

**G. [Garrett] Birkhoff and S. MacLane.** *A Survey of Modern Algebra.* (Reprint of the Third 1968 Edition.) Chelsea, New York, 1988.<sup>6</sup>

**R. Bumcrot.** *Modern Projective Geometry.* Holt, Rinehart and Winston, New York, 1969. (This could also be listed under heading **5** below).

**W. Fishback.** *Projective and Modern Geometry* (2<sup>nd</sup> Ed.). Wiley, New York, 1966.

**R. Hartshorne.** *Foundations of Projective Geometry.* Benjamin, New York, 1967.

**K. M. Hofmann and R. Kunze.** *Linear Algebra* (2<sup>nd</sup> Ed.). Prentice-Hall, Englewood Cliffs, NJ, 1971.

**G. Hochschild.** *A Second Course in Analytic Geometry.* Holden-Day, San Francisco, 1969.

**I. Kaplansky.** *Linear Algebra and Geometry – A Second Course.* Dover, New York, 2003.<sup>7</sup> (This could also be listed under heading **5** below).

**J. Murtha and E. Willard.** *Algebra and Geometry.* Holt, Rinehart and Winston, New York, 1969.

**R. Rosenbaum.** *Introduction to Projective Geometry and Modern Algebra.* Addison-Wesley, Reading, MA, 1963.

**O. Schreier and E. Sperner.** *Projective Geometry.* Chelsea, New York, 1953.

**A. Seidenberg.** *Lectures on Projective Geometry.* Dover, New York, 2006.

**J. Stillwell.** *The Four Pillars of Geometry.* (Springer) Undergraduate Texts in Mathematics. Springer-Verlag, New York-(etc.), 2005.

**R. Winger.** *Introduction to Projective Geometry.* Dover, New York, 1962.

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<sup>4</sup>MORRIS KLINE (1980–1972) is known for his books on the history and teaching of mathematics and his provocative views. A very brief description of his work and views appears in the online document [http://en.wikipedia.org/wiki/Morris\\_Kline](http://en.wikipedia.org/wiki/Morris_Kline). Kline's books contain a great deal of information and many extremely well-written passages. However, his views on several topics are highly controversial, and frequently he makes sweeping, dramatic assertions which ignore key facts or the underlying complexities of certain issues. One must be aware of these when reading his strongly negative comments about 20<sup>th</sup> century mathematics or the mathematical legacies of certain ancient or Non-Western cultures.

<sup>5</sup>Chapters X and XI are particularly relevant to these notes.

<sup>6</sup>This book has been a classic reference for undergraduate abstract algebra since the publication of the First Edition in 1941, and it was the first book of its kind in English. SAUNDERS MACLANE (1909–2005) did noteworthy work in several areas of mathematics, and he is best known for his extremely influential work on **category theory**, which is the abstract study of functions or morphisms from one mathematically structured object to another.

<sup>7</sup>IRVING KAPLANSKY (1917–2006) did important work in many different areas of algebra, including some topics with close ties to other major parts of mathematics.

#### 4. RELATED MATERIAL AT COMPARABLE LEVELS

(Most books under heading **3** also deserve a secondary classification here.)

**J. N. Cederberg.** *A Course in Modern Geometries*. (Corrected 2<sup>nd</sup> Printing of the 2<sup>nd</sup> Edition). Springer-Verlag, New York-(etc.), 2004.

**H. S. M. Coxeter.**<sup>8</sup> *The Real Projective Plane* (2<sup>nd</sup> Ed.). Cambridge Univ. Press, New York, 1955.

**H. S. M. Coxeter.** *Introduction to Geometry*. Wiley, New York, 1961.

**D. Hilbert.** *Foundations of Geometry* (2<sup>nd</sup> Ed.). Open Court Publishing, Chicago, IL, 1971.

**H. Levy.** *Projective and Related Geometries*. MacMillan, New York, 1964.

**E. E. Moïse.** *Elementary Geometry from an Advanced Standpoint* (3<sup>rd</sup> Ed.). Addison-Wesley, Reading, MA, 1991.

**M. A. Penna and R. R. Patterson.** *Projective Geometry and Its Applications to Computer Graphics*. Prentice-Hall, Englewood Cliffs, NJ, 1986.

**W. Prenowitz and M. Jordan.** *Basic Concepts of Geometry*. Xerox Publishing, Waltham, MA, 1965.

**F. S. Roberts and B. Tesman.** *Applied Combinatorics* (2<sup>nd</sup> Ed.). Prentice-Hall, Upper Saddle River, NJ, 2004.

**W. Rudin.** *Principles of Mathematical Analysis*. (3<sup>rd</sup> Ed., International Series in Pure and Applied Mathematics.) McGraw-Hill, New York, 1976.<sup>9</sup>

**P. F. Ryan.** *Euclidean and non-Euclidean Geometry: An Analytical Approach*. Cambridge Univ. Press, New York, 1986. [This book also deserves a secondary classification under heading **3**.]

**H. J. Ryser.** *Combinatorial Mathematics* (Carus Mathematical Monographs No. 14). Wiley, New York, 1963.

**J. Verdina.** *Projective Geometry and Point Transformations*. Allyn and Bacon, Boston, 1971.

**E. C. Wallace and S. F. West.** *Roads to Geometry* (3<sup>rd</sup> Ed.). Prentice-Hall, Upper Saddle River, NJ, 2003.

**A. N. Whitehead.** *The Axioms of Descriptive Geometry*. Cambridge Univ. Press, New York, 1905.

#### 5. MORE ADVANCED MATERIAL OR AT HIGHER LEVELS

(The referernces at the end of Appendix C also fit into this category.)

**A. A. Albert and R. Sandler.** *An Introduction to Finite Projective Planes*. Holt, Rinehart and Winston, New York, 1968.

**E. Artin.** *Geometric Algebra*. Wiley (Interscience), New York, 1957.

**R. Baer.** *Linear Algebra and Projective Geometry*. Dover, New York, 2005.

**G. [Garrett] Birkhoff.** *Lattice theory*. Corrected reprint of the 1967 Third Edition. American Mathematical Society Colloquium Publications, Vol. 25. American Mathematical Society, Providence, RI, 1979.

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<sup>8</sup>HAROLD SMITH MACDONALD (Donald) COXETER (1907–2003) is mainly known for his research which establishes fundamental links between classical and modern themes in geometry, his influence which is reflected by numerous classic texts on various aspect of geometry, and his interactions with famous nonmathematicians whose work had substantial mathematical content, most notably the artist MAURITS C. ESCHER (1898–1972) and the architect/visionary R. BUCKMINSTER FULLER (1895–1983).

<sup>9</sup>Since the appearance of the First Edition in 1953, this book has been a classic, frequently cited reference for the theory of functions of real variables at the undergraduate level.

- L. Blum, F. Cucker, M. Shub and S. Smale.** *Complexity and Real Computation*. Springer-Verlag, New York–etc., 1997.
- F. Buekenhout** (ed.). *Handbook of Incidence Geometry*. Elsevier Science Publishing, New York–(etc.), 1995.
- H. Crapo and G.-C. Rota.** *On the Foundations of Combinatorial Theory. Combinatorial Geometries*. MIT Press, Cambridge, MA, 1970.<sup>10</sup>
- J. Dieudonné.** *Linear Algebra and Geometry*. Hermann and Houghton Mifflin, Boston, 1968.
- H. G. Forder.** *Foundations of Euclidean Geometry*. Dover, New York, 1958.
- M. Hall.** *Theory of Groups* (3<sup>rd</sup> Ed.). AMS Chelsea, Providence, RI, 1999.
- M. Hall.** *Combinatorial Theory* (2<sup>nd</sup> Ed. Wiley, New York, 1967.
- I. N. Herstein.** *Topics in Algebra* (2<sup>nd</sup> Ed.). Wiley, New York, 1975.
- W. V. D. Hodge and D. E. Pedoe.** *Methods of Algebraic Geometry, Volumes 1 – 3* (Cambridge Mathematical Library). Cambridge Univ. Press, New York, 1994.<sup>11</sup>
- T. W. Hungerford.** *Algebra*. (Reprint of the 1974 original, Graduate Texts in Mathematics, Vol. 73.) Springer-Verlag, New York–etc., 1980.
- N. Jacobson.** *Lectures in Abstract Algebra, Vol. II: Linear Algebra*. Van Nostrand, New York, 1953.<sup>12</sup>
- W. Jenner.** *Rudiments of Algebraic Geometry*. Oxford Univ. Press, New York, 1963.
- F. Kárteszi.** *Introduction to Finite Geometries*. Elsevier, New York, 1976.
- J. L. Kelley.** *General Topology* (Graduate Texts in Mathematics). Springer-Verlag, New York–(etc.), 1975.
- C. C. Lindner and C. A. Rodger.** *Design theory*. CRC Press Series on Discrete Mathematics and its Applications. CRC Press, Boca Raton, FL, 1997.
- J. G. Oxley.** *Matroid Theory*. Oxford Univ. Press, New York, 1993.
- R. D. Schafer.** *Introduction to Nonassociative Algebras*. Dover, New York, 1996.
- A. Seidenberg.** *Elements of the Theory of Algebraic Curves*. Addison-Wesley, Reading, MA, 1968.
- J. G. Semple and G. T. Kneebone.** *Algebraic Projective Geometry* (New Edition, Oxford Classic Texts in the Physical Sciences). Oxford Univ. Press, New York, 1998.

## 6. PAPERS OR JOURNAL ARTICLES

- J. C. Baez.** *The Octonions*. Bull. Amer. Math. Soc. (2) **39** (2002), 145–205.
- R. H. Bruck and E. Kleinfeld.** *The structure of alternative division rings*. Proc. Amer. Math. Soc. **2** (1951), 878–890.
- R. H. Bruck and H. J. Ryser.** *The nonexistence of certain finite projective planes*. Canad. J. Math. **1** (1949), 88–93.
- A. Cronheim.** *A proof of Hessenberg’s Theorem*. Proc. Amer. Math. Soc. **4** (1953), 219–221.

<sup>10</sup>GIAN-CARLO ROTA (1932–1999) is mainly known for his highly influential work in combinatorics (the study of finite mathematical objects), and he also made noteworthy contributions to functional analysis and ergodic theory.

<sup>11</sup>WILLIAM VALLANCE DOUGLAS HODGE (1903–1975) discovered fundamentally important relationships involving algebraic geometry, differential geometry and partial differential equations, and his *Hodge Conjecture* on such relationships is regarded as one of the most important open questions in mathematics.

<sup>12</sup>NATHAN JACOBSON (1910–1999) made numerous fundamental contributions to the theory of abstract algebraic systems.

- J. Dieudonné.** *The historical development of algebraic geometry.* Amer. Math. Monthly **88** (1972), 827–866.
- S. Gorn.** *On incidence geometry.* Bull. Amer. Math. Soc. **46** (1940), 158–167.
- E. Kleinfeld.** *Simple alternative rings.* Ann. of Math. **58** (1953), 544–547.
- C. W. H. Lam.** *The search for a finite projective plane of order 10.* Amer. Math. Monthly **98** (1991), 305–318.
- C. W. H. Lam, L. Thiel and S. Swiercz.** *The nonexistence of finite projective planes of order 10.* Canad. J. Math. **41** (1989), 1117–1123.
- R. S. Palais.** *The classification of real division algebras.* Amer. Math. Monthly **75** (1968), 366–368.
- K. H. Parshall.** *In pursuit of the finite division algebra theorem and beyond: Joseph H. M. Wedderburn, Leonard Dickson and Oswald Veblen.* Arch. for the Internat. Hist. Science **33** (1983), 274–299.
- C. Weibel.** *Survey of non – Desarguesian planes.* Notices Amer. Math. Soc. **54** (2007), 1294–1303.
- H. Whitney.** *On the abstract properties of linear dependence.* Amer. J. Math. **57** (1935), 509–533.

## 7. UNPUBLISHED MATERIAL

**S.-S. Chern.** *Fundamental Concepts of Geometry.* Mimeographed lecture notes, University of Chicago, 1953.<sup>13</sup>

## 8. LINKS TO THE WORLD WIDE WEB

(These sites have been checked out and found to be reliable.)

<http://www.dartmouth.edu/~matc/math5.geometry/unit11/unit11.html>

[http://en.wikipedia.org/wiki/Projective\\_geometry](http://en.wikipedia.org/wiki/Projective_geometry)

<http://www.nct.anth.org.uk/basics.htm>

<http://www.math.poly.edu/~alvarez/teaching/projective-geometry/Exams/problems.html>

<http://www.math-mit.edu/~kedlaya/geometryunbound/gu-060118.pdf>

<http://robotics.stanford.edu/~birch/projective>

<http://www2.maths.ox.ac.uk/~hitchin/hitchinnotes/hitchinnotes.html>

<http://www.stolaf.edu/people/cederj/Courses.dir/bib-356/index.html#beginning>

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<sup>13</sup>SHIING-SHEN CHERN (1911–2004) was one of the leading figures in 20<sup>th</sup> century differential geometry, with research including several fundamental advances and wide ranging influence upon the development of the subject.