Math 205B
Winter 2018

UPDATED GENERAL INFORMATION — MARCH 9, 2018

Readings for Unit VII

In addition to `algtop-notes.pdf` and the corresponding exercise and solutions, here are some recommendations. An asterisk "$*$" in a file name denotes a wild card; for example, `part*.pdf` might denote files `part1.pdf` and `part2.pdf`, and similarly `filename.*` may denote different types of files with the same basic name.

`embed-graph.pdf`

Proof that a Figure 8 and a Figure Theta (θ) graph, which are homotopy equivalent, are not homeomorphic, and in fact neither is homeomorphic to a deformation retract of the other (this assertion appears in Munkres).

`brouwer.pdf`

Details of the vector-geometric input needed in the proof of the Brouwer Fixed Point Theorem.

`disk-with-holes.pdf`

Computation for the homology groups of a disk with a finite number of subdisks removed.

`fishmaze*.pdf`

Illustration of a simple closed curve in the plane for which it is not visually obvious that the complement splits into two connected subsets, and a method for deciding whether two points lie in the same or different components when the curve is well behaved.

`ahlfors.pdf`

Relation of the standard topological concept of simple connectedness to the version of Ahlfors' *Complex Analysis* textbook.

`vonKoch.pdf`

A standard example of a simple connected curve in the plane which does not have continuous tangents anywhere and whose arc length is infinite.

`vonKoch-sim.gif`

Animated illustration of the recursive construction of the curve in the preceding file.

`Jordan Curve Theorem.jpg`

Drawings of a simple closed curve in the plane which has many twists and turns but is less complicated than the one in the `fishmaze` files.

`sphere-complement.pdf`

Proof that if $A \subset S^n$ is a standardly embedded k-sphere, then $S^n - A$ is homeomorphic to $S^{n-k-1} \times \mathbb{R}^{k+1}$; for nonstandardly embedded spheres, the only information is given by the Jordan-Brouwer Theorem: The sets $S^n - A$ and $S^{n-k-1} \times \mathbb{R}^{k+1}$ have isomorphic singular homology.

horned-sphere.pdf
horned2sphere.pdf
the Alexander Sphere - YouTube.flv

Explicit classical example of a 2-sphere in S^3 whose complement is not homeomorphic to a disjoint union of two open disks; in fact, one component is not even simply connected.

graphpix4.pdf

Drawings of graphs and subgraphs arising in the results of Section VII.4.

Assignments for Unit VII

Working the exercises listed below is **strongly recommended.**

The following exercise is taken from Munkres:

p. 393: 2 (solve this using the methods of Unit VII)

The following exercises are taken from Hatcher; the page numbers refer to the numbering in the book, not the pdf file:

p. 155 *et seq.*: 4, 12, 27

The following exercises are taken from exercises05.pdf in the course directory:

2*b*, 5*a*, 9*abcd* (in (*d*), correct the misprinted conclusion to state that $\deg f$ and $\deg\left(h \circ f \circ h^{-1}\right)$ are equal), 10*a*

Here are three additional exercises:

1. Provew that $S^p \times S^q$ is not a retract of $S^p \times D^{q+1}$.

2. Suppose that X is a topological space such that every continuous mapping from X to itself has a fixed point, and suppose that the subspace inclusion mapping $j : A \to X$ is a retract. Prove that every continuous mapping from A to itself also has a fixed point. [*Hint:* If $r : X \to A$ is a one-sided inverse and $g : A \to A$ is continuous, why does the image of the composite $j \circ g \circ r$ lie in A?]

3. Suppose that C_1 and C_2 are disjoint simple closed curves in S^3, and let L be their union. Prove that the homology groups $H_q(S^3 - L)$ are isomorphic to $\mathbb{Z}$ if $q = 0$ or 2, $\mathbb{Z} \oplus \mathbb{Z}$ if $q = 1$, and 0 otherwise.

Reading assignments from solutions to exercises

The solutions to these exercises in solutions05.pdf should be read and understood at the passive level as described in an earlier posting:

2*c*, 4, 5*b*, 10*b*