Math 205B
Winter 2019

UPDATED GENERAL INFORMATION — MARCH 8, 2018

The second in-class examination

This will take place on Friday, March 15, as previously announced, and it will cover Units IV – VII with the exceptions of Sections VII.4 – VII.6 and the portion of Section IV.1 dealing with the Königsberg Bridge Problem. In addition to the practice problems given below, some recommednations are to look at the problems and solutions in the following files as well as all files whose names include one of the strings `exercises` or `solutions` or `exam` or `takehome`:

```
aabUpdate09.205B.w18.pdf
aabUpdate10.205B.w18.pdf
aabUpdate11.205B.w18.pdf
aabUpdate12.205B.w18.pdf
aabUpdate13.205B.w18.pdf
```

Here are the problems for study. Some problems on the exam will very closely resemble some of these:

1. Suppose that (X, A) is a pair of spaces such that $H_q(X, A) = 0$ for $q < n$. Prove that the map $H_q(A) \to H_q(X)$ is an isomorphism for $q < n$ and $H_n(A) \to H_n(X)$ is onto.

2. Suppose that the chain complex C_* has subcomplexes A_* and B_* such that $C_q = A_q + B_q$ for all q (but not necessarily a direct sum!), and let $j : A_* \oplus B_* \to C_*$ be the chain map which is inclusion on each summand. Formulate and prove a long exact Mayer-Vietoris sequence involving the groups $H_*(C)$, $H_*(A) \oplus H_*(B)$ and $H_*(A \cap B)$.

3. The sans-serif letters

$$\mathsf{A} \quad \mathsf{O} \quad \mathsf{P} \quad \mathsf{Q}$$

all have graph complex structures and infinite cyclic fundamental groups, but no two are homeomorphic. Give an example of another connected graph X which also has an infinite cyclic fundamental group but is not homeomorphic to any of the four examples, and prove your non-homeomorphism assertion.

4. Suppose that the chain complex C_* has subcomplexes $A_* \subset B_* \subset C_*$. Derive the following long exact sequence for the homology groups of quotient complexes:

$$\cdots H_{q+1}(C/B) \to H_q(B/A) \to H_q(C/A) \to H_q(C/B) \to H_{q-1}(B/A) \cdots$$

5. Let $P \subset \mathbb{R}^2$ be a polyhedron which is topologically a simple closed curve (*i.e.*, homeomorphic to a circle), let $y \in P$ be a point which is not a vertex, and let U be an open disk neighborhood of y which does not contain any vertices or points from edges not containing y, so that $U - P$ consists of two solid semicircular regions. Prove that the two components of $U - P$ are contained in separate components of $\mathbb{R}^2 - P$.

6. Prove that $S^2 \times \mathbb{R}$ and $S^2 \times \mathbb{R}^2$ are not homeomorphic. [*Hint:* The first space is homeomorphic to the open subset $\mathbb{R}^3 - \{0\}$. What can we say about the second?]

7. Let $P \subset \mathbb{R}^2$ be the polyhedron whose underying space is two solid squares with an edge in common. Call one of the squares $ABCD$ and the other $BEFC$. Take the simplicial decomposition determined by these five edges, the diagonals AC and BF, and the four 2-simplices into which these decompose the space, and take the alphabetical ordering of the vertices. Write down an explicit 2-chain whose boundary is a linear combination of AB, BE, EF, CF, CD, AD such that each coefficient of the boundary chain is either $+1$ or -1.