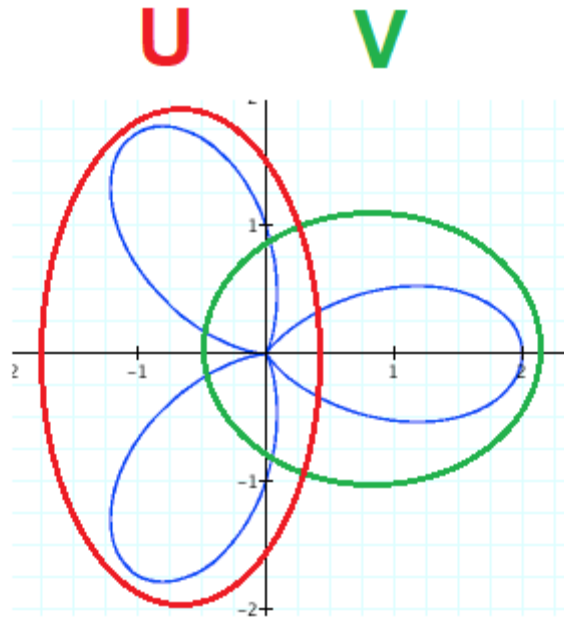# Fundamental group of a 3 – circle union

Assume we already know that the fundamental group of a 2 – circle union is a free group on 2 generators. The following argument can be modified to give an inductive argument for finding the fundamental group of an n – circle union if we know the fundamental group of a union with one fewer circle.

In the drawing, the blue curve is a model for the union of three circles which meet at a single point, and the open subsets U and V of this space are the sets of all points on the curve which are inside the red and green curves respectively. So U is the union of the two closed curves on the left with an arc on the right hand curve which contains the common point, and the union of the two closed curves is a deformation retract of U; it follows that the fundamental group of U is a free group on two generators. Similarly, V is the union of the right hand closed curve with arcs on each of the other curves, and it follows that the right hand curve is a deformation retract of V. Finally, the intersection of U and V is three open arcs which meet at a single point, and hence this intersection is contractible. By Van Kampen's Theorem, these observations imply that the fundamental group of the entire blue curve is a free group on three generators.