Proposition. If $(Y, \mathcal{E})$ is a connected graph and $p: X \to Y$ is a finite covering, then there is a graph structure $\mathcal{E}'$ on X such that p maps each edge in $\mathcal{E}'$ homeomorphically to an edge in $\mathcal{E}$.

Proof. Suppose the covering has N sheets. If E_α is an edge of Y, then $p^{-1}[E_\alpha] \to E_\alpha$ is also a finite covering. Since E_α is simply connected, it follows that $p|p^{-1}[E_\alpha]$ maps each component homeomorphically to E_α, so we may write $p^{-1}[E_\alpha] = E_{\alpha,1} \amalg \ldots \amalg E_{\alpha,N}$. To show the $E_{\alpha,j}$'s (over all α, j) form a graph decomposition, look at the intersections $E_{\alpha,i} \cap E_{\beta,j}$ when $(\alpha, i) \neq (\beta, j)$. If $\alpha = \beta$, these intersections are empty. If $\alpha \neq \beta$, then the intersection's projection to Y is at most one end point, and since $p|E_{\gamma,k}$ is 1-1, the intersection's projection is also at most one endpoint.